

RR Flux on Calabi-Yau and Partial Supersymmetry Breaking

Tomasz R. Taylor

*Department of Physics, Northeastern University
Boston, MA 02115, USA*

Cumrun Vafa

*Jefferson Laboratory of Physics
Harvard University, Cambridge, MA 02138, USA*

We show how turning on Flux for RR (and NS-NS) field strengths on non-compact Calabi-Yau 3-folds can serve as a way to partially break supersymmetry from $N = 2$ to $N = 1$ by mass deformation. The freezing of the moduli of Calabi-Yau in the presence of the flux is the familiar phenomenon of freezing of fields in supersymmetric theories upon mass deformations.

December 1999

1. Introduction

Type II strings compactified on Calabi-Yau threefolds give rise to $N = 2$ theories in 4 dimensions. The geometry of Calabi-Yau threefold and its moduli space provides a deep insight into the dynamics of $N = 2$ gauge theories. It is thus natural to ask if the simple operation of breaking supersymmetry from $N = 2$ to $N = 1$ (say by addition of mass terms) has a Calabi-Yau counterpart. If so, this may provide insight into the dynamics of $N = 1$ gauge theories.

A particular approach to breaking supersymmetry in the context of Type II compactification on Calabi-Yau threefolds was taken in [1] where Ramond-Ramond fields strengths were turned on. It was shown, however, that one either preserves all $N = 2$ supersymmetries (and freeze the moduli of Calabi-Yau to make it correspond to singular limits such as the conifold) or one breaks the supersymmetry completely. Furthermore it was argued in [2] that this is a general result.

On the other hand it was found in [3] that in the context of $N = 2$ quantum field theories it is possible to add $N = 2$ FI terms, and break the supersymmetry to $N = 1$. These constructions were generalized to the local case in [4] (see also [5]). There seemed, therefore, to exist a conflict between the results coming from considerations of type II compactifications on Calabi-Yau threefolds which suggested finding $N = 1$ supersymmetric theories by turning on fluxes is not possible, whereas field theory arguments suggested that some such deformations should be possible.

We will see in this paper that indeed we can obtain partial supersymmetry breaking by considering non-compact Calabi-Yau manifolds with fluxes turned on. The way this avoids the no-go theorem in [2] is by taking a certain decompactification limit, which renders some fields non-dynamical. In other words, it would have corresponded to a theory with no supersymmetric vacua in the compact situation, and where it not for making some fields non-dynamical, we could not have obtained partial supersymmetry breaking. However, as far as geometric engineering of $N = 1$ theories are concerned the non-compactness of the Calabi-Yau is a perfectly acceptable condition, and this is already the case for geometric engineering of $N = 2$ theories.

The organization of this paper is as follows: In section 2 we show how supersymmetry can be partially broken by considering a simple generalization of models of [3] where we include two $N = 2$ vector multiplets. In section 3 we consider type II compactifications on Calabi-Yau threefolds and show why turning on RR fluxes (and in addition NS flux H

for type IIB) is equivalent to turning on FI terms in the $N = 2$ supersymmetric theory. We also review the no-go theorem of [2] and show how it may be avoided in certain non-compact limits. In section 4 we briefly review aspects of $N = 2$ geometric engineering and show how RR flux can give mass to the adjoint field breaking the theory to $N = 1$ and freezing some of the Calabi-Yau moduli.

2. Partial Supersymmetry Breaking and Mass Generation

In this section, we present a simple generalization of the model discussed in [3] which exhibits partial supersymmetry breaking with mass generation for $N = 1$ multiplets. It involves two $N = 2$ vector multiplets, S and A , with the prepotential

$$\mathcal{F}(S, A) = \frac{i\gamma}{2}S^2 + \frac{1}{2}SA^2, \quad (2.1)$$

where γ is a real constant, and with the superpotential (which in general can be taken to be a linear combination of “periods”)

$$W = eS + m\mathcal{F}_S = (e + im\gamma)S + \frac{m}{2}A^2, \quad (2.2)$$

where $e = e_1 + ie_2$ and $m = m_1 + im_2$ are *complex* constants and $\mathcal{F}_S = \partial\mathcal{F}/\partial S$. The corresponding Lagrangian is $N = 2$ supersymmetric. The constants e and m correspond to $N = 2$ electric and magnetic Fayet-Iliopoulos terms, respectively. In the manifestly $N = 2$ supersymmetric notation of Ref.[3]:

$$\text{Re}\vec{E} = (e_1 \ e_2 \ 0), \quad \vec{M} = (m_1 \ m_2 \ 0). \quad (2.3)$$

The superpotential of Eq.(2.2) gives rise to the following potential for the scalars $S = \alpha + i\sigma$ and $A = b + ia$:

$$V = \frac{|ea - \gamma mb|^2}{\gamma(\gamma\sigma - a^2)}. \quad (2.4)$$

In the above equation, we neglected an irrelevant, additive constant term. The potential has a zero-value minimum at $a = b = 0$. For generic values of e and m , both $N = 2$ supersymmetries are broken spontaneously. There are, however, two special configurations of these parameters:

$$e \pm im\gamma = 0, \quad (2.5)$$

for which supersymmetry is broken partially to $N = 1$. In this case, both scalars a and b , as well as the fermionic component of the $N = 1$ chiral multiplet A acquire an equal mass

of $|m|/\sigma$. The simplest way to prove formally that such a partial breaking does indeed occur is to follow the method of [3] and examine the supersymmetry variations of fermions. In this way, one can identify the $N = 2 \rightarrow N = 1$ goldstino as one of the two fermionic components (gauginos) of the S multiplet. In fact, the full $N = 2$ vector multiplet S and the $N = 1$ vector component of A remain massless while the $N = 1$ chiral multiplet A acquires a mass.

The above model can be generalized to more complicated prepotentials, of the form

$$\mathcal{F}(S, A) = f(S) + \frac{1}{2}SA^2. \quad (2.6)$$

As in the previous case, the potential has a minimum at $A = 0$. However, there is also another minimization, with respect to S , which yields two solutions

$$e + m\mathcal{F}_{SS} = 0 \quad \text{or} \quad e + m\bar{\mathcal{F}}_{\bar{S}\bar{S}} = 0, \quad (2.7)$$

similar to (2.5). It is easy to see that the above equation is exactly the condition for partial supersymmetry breaking. Hence we conclude that an $N = 1$ supersymmetric vacuum exists also in the general case. In particular, the mass $|m|/\sigma$ is generated again for the $N = 1$ chiral multiplet A .

So far we have been discussing the case of global $N = 2$ supersymmetry. In the context of string theory we of course have local $N = 2$ supersymmetry. In such a case to obtain $N = 2$ global limit we have to take some particular limit, where gravity decouples, say by taking in the type II context weak limit of string coupling constant, and perhaps some other limits for other fields. In this context we can break $N = 2$ to $N = 1$ in an even simpler way. Set $\gamma = 0$, so that the prepotential is just

$$\mathcal{F} = \frac{1}{2}SA^2$$

This would have given a singular kinetic term for S in the global case, but it is perfectly fine in the local case. We can think of S for example as the “heterotic string coupling constant”. We now turn on FI term $\alpha\mathcal{F}_S + \beta\mathcal{F}_A$. We take the limit where the vev of S becomes large (i.e. weak coupling heterotic string limit). In this limit S becomes non-dynamical. And the superpotential term $W = \frac{1}{2}m(A')^2$ (where A' is related to A by a shift) simply gives mass to the scalar A' , breaking $N = 2$ to $N = 1$. It is this realization of partial supersymmetry breaking that we will find applicable in the Calabi-Yau context later in this paper.

3. Type IIB on Calabi-Yau 3-fold with H -flux

Consider compactification of type IIB on a Calabi-Yau threefold. We would like to consider turning on flux for NS and R threeform field strengths H^{NS} and H^R . This is a case already considered in [2] following the work of [1] and more recently from the viewpoint of F-theory in [6,7]. The theory has $h^{2,1}$ vector multiplets and $h^{1,1} + 1$ hypermultiplets in addition to the $N = 2$ gravitational multiplet, where $h^{p,q}$ denotes Hodge numbers of Calabi-Yau. The relevant modification to the effective action due to turning on H -flux is in interactions with the vector multiplets. Let Ω denote the holomorphic threeform on the Calabi-Yau. We write the effective Lagrangian we obtain in 4 dimensions in an $N = 1$ supersymmetric framework. The net effect of turning on H -flux is to add a superpotential of the form

$$W = \int \Omega \wedge (\tau H^{NS} + H^R) \quad (3.1)$$

in the 4-dimensional effective theory, where τ denotes the complexified coupling constant of type IIB strings. Note that H^{NS} and H^R are dual to some integral 3-cycles C_{NS} and C_R and the above formula can also be written as

$$W = \int_{\tau C_{NS} + C_R} \Omega$$

To see how (3.1) arises note that if we consider a five brane (NS or R) wrapped around a 3-cycle C in the Calabi-Yau, it corresponds to a domain wall in 3+1 dimensional theory, whose BPS bound for tension should be given by ΔW across the domain wall. On the other hand the tension of the 5-brane should be $\int_C \Omega$ (times τ in the case of NS 5-brane). Since the 5-brane wrapped around C changes the H flux across the domain wall by a 3-form dual to the C cycle we see that this gives the expected change ΔW . This argument was discussed in [6] in the context of F-theory on 4-folds, and type IIB on Calabi-Yau 3-folds is a special case of it.

We can also write (3.1) explicitly if we choose a basis for $H_3(M, \mathbf{Z})$, given by (A^Λ, B_Σ) , $\Lambda, \Sigma = 0, \dots, h^{2,1}$, with $A^\Lambda \cap A^\Sigma = B_\Lambda \cap B_\Sigma = 0$ and $A^\Lambda \cap B_\Sigma = \delta_\Sigma^\Lambda$. Sometimes we refer to A^Λ as the electric cycles and B_Σ as the magnetic cycles. This clearly is a basis dependent definition. Let

$$X^\Lambda = \int_{A^\Lambda} \Omega \quad \mathcal{F}_\Sigma = \int_{B_\Sigma} \Omega$$

Moreover denote the dual 3-cycle to the H-fluxes by

$$\tau C^{NS} + C^R = e_\Lambda A^\Lambda + m^\Lambda B_\Lambda$$

where

$$C^{NS} = e_{\Lambda}^1 A^{\Lambda} + m^{1\Lambda} B_{\Lambda} \quad C^R = e_{\Lambda}^2 A^{\Lambda} + m^{2\Lambda} B_{\Lambda} , \quad (3.2)$$

and the complex vectors e and m are defined as:

$$e_{\Lambda} = e_{\Lambda}^1 \tau + e_{\Lambda}^2 \quad m^{\Lambda} = m^{1\Lambda} \tau + m^{2\Lambda} . \quad (3.3)$$

The superpotential (3.1) can be written explicitly as

$$W = \int_{C^R} \Omega + \tau \int_{C^{NS}} \Omega = e_{\Lambda} X^{\Lambda} + m^{\Lambda} \mathcal{F}_{\Lambda} \quad (3.4)$$

As is well known there is a prepotential $\mathcal{F}(X)$, a homogeneous function of weight 2 in X in terms of which

$$\mathcal{F}_{\Lambda} = \partial_{\Lambda} \mathcal{F} .$$

Thus the FI terms are realized by H fluxes in type IIB string compactification on Calabi-Yau threefolds.

3.1. Type IIA version

The same analysis can be done in the type IIA language (for the case of type IIA on Calabi-Yau 4 folds see [8]). In fact mirror symmetry already tells us what the story will be in the type IIA case. The story is much simpler in the context of just turning on the H^R flux. In this case the mirror corresponds to turning on F^2, F^4 and F^6 fluxes which are dual to 4, 2 and 0 cycles on Calabi-Yau 3-fold. The analog of (3.1) is now

$$W = N_0 + \int_{C_2} k + \int_{C_4} k^2$$

where k denotes the Kähler class on the Calabi-Yau threefold and N_0 denotes the quantum of F^6 flux. The above formula receives world sheet instanton correction as is well known, and in fact by mirror symmetry one can recover the instanton corrected superpotential W on the Calabi-Yau.

3.2. Scalar Potential

Our next step is to obtain the scalar potential corresponding to the superpotential (3.4). We would like to maintain a manifest $N = 2$ supersymmetry, however this is not possible in the locally supersymmetric case because the superpotential is a genuinely $N = 1$ quantity. Instead of turning to the fully-fledged $N = 2$ supersymmetric formalism [9] (like in Ref.[2]), we can try to obtain the potential by compactifying the 10-dimensional action. Alas, this is not so simple in view of the absence of a fully covariant, off-shell formulation of type IIB supergravity. The best we can do is to start from the “non-self-dual” (NSD) action [10] employing a 4-form field strength which is not self-dual. The equations of motion of IIB supergravity follow from the NSD action after imposing the self duality constraint at the level of field equations. Using the NSD action to determine the scalar potential is somewhat questionable, nevertheless it is interesting to compare the result with the superpotential (3.4). In fact, this method will provide an independent derivation of (3.4).

In order to parameterize the H -fluxes, we will use the $H^3(M, \mathbf{Z})$ basis $(\alpha_\Lambda, \beta^\Sigma)$, dual to the (A^Λ, B_Σ) basis of $H_3(M, \mathbf{Z})$, with $\int \alpha_\Lambda \wedge \beta^\Sigma = \delta_\Lambda^\Sigma$, $\int \alpha_\Lambda \wedge \alpha_\Sigma = \int \beta^\Lambda \wedge \beta^\Sigma = 0$. The fluxes can be written as

$$H^{NS} = e_\Lambda^1 \beta^\Lambda + m^{1\Lambda} \alpha_\Lambda, \quad H^R = e_\Lambda^2 \beta^\Lambda + m^{2\Lambda} \alpha_\Lambda. \quad (3.5)$$

In the presence of the fluxes, the 10-dimensional kinetic terms give rise to the potential:

$$V = (2\text{Im}\tau)^{-1} \int (\tau H^{NS} + H^R) \wedge *(\bar{\tau} H^{NS} + H^R). \quad (3.6)$$

The integration over the Calabi-Yau manifold can be performed by using standard properties of $(\alpha_\Lambda, \beta^\Lambda)$ basis (see e.g. [11,12]), with the result

$$V = -(2\text{Im}\tau)^{-1} [m(\text{Im}\mathcal{N})\bar{m} + (e + m\text{Re}\mathcal{N})(\text{Im}\mathcal{N})^{-1}(\bar{e} + \bar{m}\text{Re}\mathcal{N})], \quad (3.7)$$

where \mathcal{N} is the period matrix [12] while e and m are the complex vectors defined in (3.3). The potential can be rewritten as

$$V = -(2\text{Im}\tau)^{-1} [(e + m\bar{\mathcal{N}})(\text{Im}\mathcal{N})^{-1}(\bar{e} + \bar{m}\mathcal{N})] + m \times e, \quad (3.8)$$

where the constant term

$$m \times e \equiv m^{1\Lambda} e_\Lambda^2 - m^{2\Lambda} e_\Lambda^1. \quad (3.9)$$

As we will discuss in the next subsection m and e should be chosen so that $m \times e$ is zero (for cancellation of 3-brane tadpoles), which we will assume is the case.

In order to relate the above potential with the superpotential (3.4), we first use the identity [12]:

$$e^{-K(z, \bar{z})} (\text{Im} \mathcal{N})^{-1\Lambda\Sigma} = -2\bar{X}^\Lambda X^\Sigma - 2D_i X^\Lambda G^{i\bar{j}} D_{\bar{j}} \bar{X}^\Sigma, \quad (3.10)$$

where K is the Kähler potential of the $N = 2$ vector multiplet moduli z_i , $i = 1, \dots, h^{2,1}$, and $G^{i\bar{j}}$ is the inverse metric on the vector moduli space. The above expression contains the Kähler covariant derivatives:

$$D_i X^\Lambda = (\partial_i + K_i) X^\Lambda. \quad (3.11)$$

By using the relations

$$\mathcal{N}_{\Lambda\Sigma} X^\Sigma = \mathcal{F}_\Lambda, \quad \bar{\mathcal{N}}_{\Lambda\Sigma} D_i X^\Sigma = D_i \mathcal{F}_\Lambda \quad (3.12)$$

we can rewrite the potential as

$$V = e^{[K(z, \bar{z}) + \tilde{K}(\tau, \bar{\tau})]} [G^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} + \tilde{G}^{\tau\bar{\tau}} D_\tau W D_{\bar{\tau}} \bar{W}], \quad (3.13)$$

where W is the superpotential (3.4). The dilaton Kähler potential is $\tilde{K}(\tau, \bar{\tau}) = -\ln[(\tau - \bar{\tau})/2i]$ and accordingly,

$$D_\tau W = (\partial_\tau + \tilde{K}_\tau) W, \quad \tilde{G}^{\tau\bar{\tau}} = \tilde{K}_{\tau\bar{\tau}}^{-1} = -(\tau - \bar{\tau})^2. \quad (3.14)$$

Eq.(3.13) is very similar to the standard $N = 1$ supergravity formula for the potential. However, it is not exactly the same: for instance, the $-3|W|^2$ term is missing. This apparent discrepancy has a simple explanation. Although the superpotential does not depend on hypermultiplets, except on the dilaton τ , the potential receives contributions from the Kähler covariant derivatives¹ with respect to chiral components of all hypermultiplets, including the Calabi-Yau volume etc. All these contributions are proportional to $|W|^2$ and must cancel the $-3|W|^2$ term. The coefficient -3 is related to the fact that the 4d coupling is rescaled by the volume of the internal Calabi-Yau threefold.

Now we consider the rigid supersymmetry limit of (3.13) and (3.4). The Weyl rescaling of the metric that restores the Planck mass M_{Pl} in the action introduces the factors M_{Pl}^2 in

¹ For simplicity, we assume here that the quaternionic hypermultiplet manifold is Kähler.

front of the Ricci scalar R and scalar kinetic terms; the potential acquires a factor of M_{Pl}^4 . Gravity decouples in the $M_{Pl} \rightarrow \infty$ limit and the only scalars surviving as dynamical fields are those with the Kähler metric $\sim M_{Pl}^{-2}$; all other scalars “freeze” and can be treated as constant parameters. In order to recover the globally supersymmetric models of the type discussed in Section 2 and in Ref.[3] from type IIB theory with H -fluxes, we scale the holomorphic sections as $(X^0, \mathcal{F}_0) \sim 1$ and $(X^\Lambda, \mathcal{F}_\Lambda) \sim M_{Pl}^{-1}$, $\Lambda > 0$. The Kähler potential $K(z, \bar{z})$ scales then as M_{Pl}^{-2} , so the vector moduli survive as dynamical fields in the $M_{Pl} \rightarrow \infty$ limit while τ and other hypermultiplets decouple and can be treated as (complex) parameters. Furthermore, we set $e_0 = m^0 = 0$ and adjust the remaining Fayet-Iliopoulos parameters so that the superpotential (3.4) scales as M_{Pl}^{-3} . In this way [9], we obtain from (3.13) a finite potential corresponding to the rigid superpotential $W = e_i z^i + m^i \mathcal{F}_i$ [3]. The procedure of taking the rigid limit can be further refined to treat some vector moduli in a special way (such as the S field discussed in the previous section), in order to freeze them in a way similar to τ . This will be useful in the context of geometric engineering of $N = 1$ theories, to be discussed in the next section.

3.3. Supersymmetric Vacua

Now we analyze supersymmetric solutions with superpotential given by (3.4). The condition for getting a supersymmetric solution with R^4 background in this context has been studied by [2] with the conclusion that either there are no supersymmetric vacua or that the $N = 2$ is preserved at the vacua. In particular no $N = 1$ supersymmetric vacua were found in this way. Let us review these results in the $N = 1$ setup that we are considering. The condition for finding supersymmetric vacua in R^4 background is that

$$W = dW = 0$$

where dW denotes the derivative of W with respect to all chiral fields. In the context of compact Calabi-Yau, considered in [2], turning on both H^{NS} and H^R can preserve supersymmetry only if

$$\int H^{NS} \wedge H^R \sim m \times e = 0. \quad (3.15)$$

Otherwise these fluxes induce anti-3-brane charge in the uncompactified spacetime (proportional to $\int H^{NS} \wedge H^R$) and to cancel it we will necessarily break supersymmetry. If H^{NS} and H^R satisfy (3.15) then we can choose both of them be dual to some A-cycles (i.e.

m=0). If we denote the dual three cycles by $N_1 A_1$ and $N_2 A_2$ with periods $\int_{A_i} \Omega = X_i$ the superpotential will take the form,

$$W = N_1 \tau X_1 + N_2 X_2.$$

The condition that $W = dW = 0$ in terms of physical fields $z_i = X_i/X_0$, is equivalent to $dW = 0$ in terms of the X_i variables. Since X_1 and X_2 are independent fields, we see that condition $dW/dX_1 = dW/dX_2 = 0$ has no solutions, and so supersymmetry is completely broken.

So we see that if we consider smooth Calabi-Yau manifolds there are no supersymmetric solutions. However, near singular Calabi-Yau manifolds the low energy effective Lagrangian description breaks down and one could have additional light fields. The particular case of conifold was studied in [1]. In that case, say we have a vanishing A cycle, and we turn on a flux dual to that cycle. This means we have a superpotential

$$W = \alpha a$$

where $a = \int_A \Omega$ and $\alpha = n_1 + n_2 \tau$. Let us set $n_2 = 0$. If A is shrinking we have in addition a light wrapped D3 brane which in $N = 1$ language corresponds to chiral fields ϕ and $\tilde{\phi}$ with charge ± 1 respectively under the $U(1)$ gauge field whose supersymmetric scalar partner has vev $\langle \phi \rangle = a$. So the actual Lagrangian superpotential should be modified to

$$W = \alpha a + a \phi \tilde{\phi}$$

Now the condition that $W = dW = 0$ has a solution and is given by

$$a = 0 \quad \phi \tilde{\phi} = -\alpha$$

In fact it is possible to check that this actually preserves the full $N = 2$ supersymmetry. Even though some other singularities of Calabi-Yau manifolds have also been considered in [2] none has been shown to lead to $N = 1$ unbroken supersymmetry (though a full no-go theorem is not available in this context).

3.4. How to obtain $N = 1$ supersymmetry?

It thus seems difficult to obtain an $N = 1$ supergravity solution with H -flux turned on for compact Calabi-Yau manifolds. How could we possibly relax some conditions to make this possible? The hint comes from considering $N = 1$ supersymmetric Yang-Mills theories. In these cases one expects to have a mass gap with some number of vacua $c_2(G)$ given by the dual Coxeter number of the group. Moreover one can assign a superpotential to each vacuum given by

$$W_k = \omega_k \exp(-S/c_2(G))$$

where $S = 1/g^2$ and ω_k is an $c_2(G)$ root of unity. The meaning of this superpotential is that the domain walls stretched going from one vacuum to the other will have a central terms for their tension given by the difference of the corresponding values of the superpotential. In the usual $N = 1$ Yang-Mills the coupling constant S is not a field but a parameter. But we can actually promote it to a chiral field whose vev is given by the coupling constant. If we do that, then we will also have to consider $dW_k/dS = 0$ for finding supersymmetric ground states, otherwise we would get a positive energy given by

$$g^{S\bar{S}} |\partial_S W_k|^2$$

where $g^{S\bar{S}}$ is the inverse to the Kähler metric for S . However, $dW_k/dS = 0$ has no solutions, which means that we have no supersymmetric vacuum (or any vacuum in this case). But clearly we can embed the usual $N = 1$ gauge theory in this theory, simply by taking the kinetic term for S field to be very large and thus effectively freezing it (this corresponds to making $g^{S\bar{S}}$ vanishing and thus giving no energy to the vacuum). Note in this case that even if S is treated as a parameter the vacuum has an energy and so the supersymmetric background makes sense if we decouple the gravity, by taking $M_{Pl} \rightarrow \infty$. This is in fact how we will generate $N = 1$ QFT's by turning on H -fluxes; namely, as we will see later, the field S will play the role of an extra field, whose dynamics we will freeze in the limit of interest and concentrate on a decoupling limit where gravity is irrelevant. This will in particular avoid the no-go theorem of [2] for obtaining $N = 1$ solutions .

4. Geometric Engineering for $N = 2$ Theories and Their Partial Breaking to $N = 1$

In preparation of our discussion for turning on RR-fluxes and breaking $N = 2$ theories to $N = 1$ we first review some relevant aspects of geometric engineering for $N = 2$ gauge theories, in the context of type IIA compactification on Calabi-Yau threefold and its type IIB mirror (see [13-16] for more detail). Instead of being general, consider a simple example: Let us review how the $SU(2)$ Yang-Mills is geometrically engineered: We consider in type IIA a local CY geometry where a P^1 is fibered over another P^1 . The simplest possibility is $P_f^1 \times P_b^1$. In the limit the P_f^1 goes to zero we obtain an enhanced $SU(2)$ gauge symmetry from the A_1 singularity. To suppress the gravity effects we take $g_s \rightarrow 0$. The area of P_b^1 , S , determines the coupling constant for the $SU(2)$ gauge theory:

$$S = \frac{1}{g^2}$$

We can identify the size of P_f^1 , a , with the vev of a scalar in the adjoint of $SU(2)$; more precisely, we have the classical relation that

$$a^2 = \langle Tr \phi^2 \rangle.$$

One considers the regime where S is large and a is small. This is the same as taking a finite size P_b^1 and zero size P_f^1 in the string frame in the limit where $g_s \rightarrow 0$ (the effective mass of $D2$ branes is given by $a = a'/g_s$ and $S = S'/g_s$ where the a' and S' denote the volume of the P^1 's in the string frame). In this limit the field S becomes non-dynamical and plays only the role of a parameter in the field theory. The $N = 2$ prepotential in this case has the following structure

$$\mathcal{F} = \frac{1}{2} S a^2 + a^2 \log a^2 + \sum_n c_n a^{2-4n} \exp(-nS) \quad (4.1)$$

for some constants c_n . Note that S can be absorbed into a redefinition of a . In fact this is the limit one is taking in the geometry, namely we take $a \rightarrow 0$ and $S \rightarrow \infty$ keeping the combination $a^4 \exp S$ fixed. The way this expression is obtained is to use mirror symmetry to compute the worldsheet instanton corrections in this type IIA background by relating it to complex structure variation in a type IIB background. Physically, the net result is that the apparent instanton corrections in (4.1) can be related to one loop corrections to the prepotential summed over all $D2$ branes wrapped around the $P^1 \times P^1$ geometry. In other words

$$\mathcal{F} = S a^2 + \sum_{BPS\ m} m^2 \log m^2$$

and the rich structure of the instanton corrections gets mapped to an intricate structure of wrapped BPS $D2$ branes in this geometry, in the limit we are taking.

4.1. *Adding the Flux and breaking $N = 2 \rightarrow N = 1$*

Now we are ready to add the flux. We choose the flux to be an RR 2-form flux in the direction dual to the 2-cycle P_b^1 . This means according to our discussion in section 3, generating a superpotential given by

$$W = \frac{\partial \mathcal{F}}{\partial S}$$

In the context of type IIB it means turning on a specific RR H-flux dual to the 3-cycle representing S . In that context $\partial \mathcal{F} / \partial S$ is a classical computation of periods. Now, it has been shown [17] that

$$\frac{\partial \mathcal{F}}{\partial S} = \text{constant} \cdot u$$

where $u = \langle \text{tr} \Phi^2 \rangle$, and the classical result $u = a^2$ receives quantum correction exactly as given by $u = \text{constant} \cdot \frac{\partial \mathcal{F}}{\partial S}$. In particular, not just a , but also u itself is among the periods of the type IIB Calabi-Yau geometry (this was a crucial fact for extracting gauge theory implication of type IIB geometry [13]). We thus have a superpotential

$$W = \text{constant} \cdot u$$

This can be viewed in the gauge theory language as a mass deformation (giving mass to the scalar partner Φ of the vector multiplet) breaking $N = 2 \rightarrow N = 1$. To find whether there are any supersymmetric vacua one will have to analyze it, exactly as was done originally in the field theory context by Seiberg and Witten [18]. Namely one finds near the points where there is a massless dyon, a modification of the superpotential, by including the light states. In this case we get (somewhat analogous to the conifold case)

$$W = mu + (a_D - a_0)\phi\tilde{\phi}$$

and one finds an $N = 1$ supersymmetric solution

$$\phi\tilde{\phi} = m \frac{\partial u}{\partial a_D} \quad a_D = a_0$$

In the type IIB context this corresponds to freezing some complex moduli of Calabi-Yau threefold and rendering some other moduli non-dynamical.

4.2. Generalizations

It is clear that this generalizes to a large number of $N = 2$ gauge models engineered in [16] (at least for the cases where the beta function is not zero). Just as was done above for the $SU(2)$ case, one turns on in the type IIA context independent RR 2-form flux in the bases whose sizes control the coupling constants of various gauge groups. By going over to its type IIB mirror, the RR flux turns into a particular H^R flux, which again serve to freeze the moduli in the type IIB side, exactly as was done for the $SU(2)$ case above.

Acknowledgements We have benefited from discussions with S. Ferrara, K. Hori, M. Porrati and A. Strominger. The work of T.R.T. was supported in part by NSF grant PHY-99-01057 and that of C.V. was supported in part by NSF grant PHY-98-02709.

References

- [1] J. Polchinski and A. Strominger, *Phys. Lett. B* 388 (1996) 736.
- [2] J. Michelson, *Nucl. Phys. B* 495 (1997) 127.
- [3] I. Antoniadis, H. Partouche and T.R. Taylor, *Phys. Lett. B* 372 (1996) 83; I. Antoniadis and T.R. Taylor, *Fortsch. Phys.* 44 (1996) 487; H. Partouche and B. Pioline, *Nucl. Phys. Proc Suppl.* 56B (1997) 322.
- [4] S. Ferrara, L. Girardello and M. Porrati, *Phys. Lett. B* 376 (1996) 275; M. Porrati, *Nucl. Phys. Proc. Suppl.* 55B (1997) 240.
- [5] J. Bagger and A. Galperin, *Phys. Rev. D* 55 (1997) 1091; M. Roček and A. A. Tseytlin, *Phys. Rev. D* 59 (1999) 106001.
- [6] S. Gukov, C. Vafa and E. Witten, hep-th/9906070.
- [7] K. Dasgupta, G. Rajesh and S. Sethi, *JHEP* 9908 (1999) 023.
- [8] S. Gukov, hep-th/9911011.
- [9] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D'Auria, S. Ferrara, P. Fré and T. Magri, *J. Geom. Phys.* 23 (1997) 111.
- [10] E. Bergshoeff, H.J. Boonstra and T. Ortin, *Phys. Rev. D* 53 (1996) 7206.
- [11] H. Suzuki, *Mod. Phys. Lett. A* 11 (1996) 623.
- [12] A. Ceresole, R. D'Auria and S. Ferrara, *Nucl. Phys. Proc. Suppl.* 46 (1996) 67.
- [13] S. Kachru, A. Klemm, W. Lerche, P. Mayr and C. Vafa, *Nucl. Phys. B* 459 (1996) 537.
- [14] A. Klemm, W. Lerche, P. Mayr, C. Vafa and N. Warner, *Nucl. Phys. B* 477 (1996) 746.
- [15] S. Katz, A. Klemm and C. Vafa, *Nucl. Phys. B* 497 (1997) 173.
- [16] S. Katz, P. Mayr and C. Vafa, *Nucl. Phys. B* 497 (1997) 173.
- [17] M. Matone, *Phys. Lett. B* 357 (1995) 342.
- [18] N. Seiberg and E. Witten, *Nucl. Phys. B* 431 (1994) 484.