

Generalized Conformal Symmetry and Oblique AdS/CFT Correspondence for Matrix Theory

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Abstract. The large N behavior of Matrix theory is discussed on the basis of the previously proposed generalized conformal symmetry. The concept of ‘oblique’ AdS/CFT correspondence, in which the conformal symmetry involves both the space-time coordinates and the string coupling constant, is proposed. Based on the explicit predictions for two-point correlators, possible implications for the Matrix-theory conjecture are discussed.

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The Matrix-theory conjecture [1] requires us to investigate the dynamics of D-particles described by the supersymmetric Yang-Mills matrix quantum mechanics in the large N limit. Unfortunately, very little is known as for the relevant large N behaviors of the matrix quantum mechanics. In the present talk, I shall discuss the large N limit of Matrix theory by extending the AdS/CFT correspondence to the matrix quantum mechanics using the previously proposed generalized conformal symmetry as a guide.

The paper is organized into three parts. In the first part, I begin by briefly recalling the Matrix-theory conjecture and review the so-called DLCQ interpretation at finite N . The latter interpretation will be used as an intermediate step for our later arguments. Then in the second part, after a brief discussion on the generalized conformal symmetry [2, 3] from the point of view of the AdS/CFT correspondence, I introduce the notion of ‘oblique’ AdS/CFT correspondence for nonconformal D0-branes. In the third part, I discuss the results of the harmonic analysis of supergravity fluctuations around the D-particle background and its predictions for the two-point correlators of Matrix theory operators in the large N limit, based on our recent work [4] which contains, to my knowledge, the first extensive computation of the correlators for *dilatonic* case based on the AdS/CFT correspondence. I then propose to interpret the anomalous large N scaling behavior found from this analysis as an indication of a screening mechanism which may reconcile the holographic growth of the transverse size with 11 dimensional boost invariance.

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1. Matrix-theory conjecture and the DLCQ interpretation

The basic assumption of Matrix theory [1] is that the low-energy effective super Yang-Mills theory of D-particles in type IIA superstring theory is the exact description of ‘M-theory’ in the infinite-momentum limit where the total 11th dimensional momentum P_{10} becomes infinite:

$$P_{10} = N/R \rightarrow \infty, \quad (1.1)$$

where $R = g_s \ell_s$ is the radius of the 11th dimension, which is compactified to a circle, and N is the number of D-particles. The D-particles are now interpreted as partons as the basic constituents of M-theory. The effective action is

$$S = \int dt \operatorname{Tr} \left(\frac{1}{2g_s \ell_s} D_t X_i D_t X_i + i\theta D_t \theta + \frac{1}{4g_s \ell_s^5} [X_i, X_j]^2 - \dots \right). \quad (1.2)$$

In the infinite-momentum limit, the 11th momentum and the Hamiltonian can be identified with the light-like momentum and the light-like Hamiltonian, respectively.

$$P_{10} \rightarrow P_- = N/R, \quad H \rightarrow P^+ = -2P^- = R \operatorname{Tr} h = \frac{N}{P_-} \operatorname{Tr} h, \quad (1.3)$$

$$h = \frac{1}{2} \Pi^2 - \frac{1}{4\ell_P^6} [X^i, X^j]^2 + \frac{1}{2\ell_P^3} [\theta_\alpha, [X^k, \theta_\beta]] \gamma_{\alpha\beta}^k, \quad (1.4)$$

where we have introduced the 11 dimensional Planck length $\ell_P = g_s^{1/3} \ell_s$. For any finite and fixed R , the infinite-momentum limit requires the large N limit, $N \rightarrow \infty$. The (super) Galilean symmetry of this system is indeed consistent with the infinite-momentum frame interpretation.

Obviously, the infinite-momentum limit can also be achieved for fixed N if we take the limit of small compactification circle, $R \rightarrow 0$, namely the type IIA limit. If we boost back to a *finite* $P'_- = N/R'$, this defines a theory in which the system is effectively compactified along the light-like direction $x^- \sim x^- + 2\pi R'$. This is nothing but the discrete light-cone interpretation (DLCQ), proposed by Susskind [5] and elucidated in [6]. The basic assumption for this interpretation is that the dynamics is invariant under the boost along the 11th direction for finite and fixed N in contrast to the original BFSS conjecture which assumes that the boost is associated with a change of N for fixed R . In fact, if we fix the 11 dimensional Planck length, the Hamiltonian and P_- transform as desired, $P^- \rightarrow \rho P^-$ under the change, $R \rightarrow \rho R \leftrightarrow P_- \rightarrow \rho^{-1} P_-$, of the compactification radius. This may be called as the ‘kinematical’ boost transformation, contrasting to the ‘dynamical’ boost of the BFSS conjecture. Usually, the limit $R \rightarrow 0$ ($R \ll \ell_P$) which corresponds to the weak-coupling limit $g_s \rightarrow 0$ is regarded as justifying the matrix model, since the characteristic length scale $\ell \sim \ell_P$ of the model is now much smaller $\ell_P \ll \ell_s$ than the string scale, and hence the massive string modes of the open strings stretched among D-particles are decoupled as long as we are interested only in the energy range much smaller than the characteristic string unit $E(\sim P^+) \ll \ell_s^{-1}$. The two conditions, $\ell \sim \ell_P$ and $E \ll \ell_s^{-1}$, which might look naively contradictory to each other, can be

compatible since now the effective mass $m \sim 1/g_s \ell_s$ in 10 dimensional space-time is very heavy and the characteristic velocity of D-particles is given by $v \sim g_s^{2/3}$.

However, it should be emphasized that the DLCQ interpretation cannot be regarded as a ‘proof’ that the matrix model is consistent with supergravity in the long-distance limit. In the limit of small compactification radius, 11 dimensional supergravity reduces to 10 dimensional type IIA supergravity. Only natural justification of type IIA supergravity relying upon the dimensional arguments is the zero-slope limit in which $\ell \gg \ell_s$ by keeping the 11 dimensional Planck length $\ell_{10} = g_s^{1/4} \ell_s$ fixed. This limit clearly exceeds the range of the DLCQ region $\ell \ll \ell_s$. Note that the low-energy limit $\ell \gg \ell_P$ in the sense of 11 dimensions can be used as a criterion *only* in the *decompactification* (*i.e.*, strong coupling) limit $R \rightarrow \infty$ for fixed ℓ_P .

In spite of this apparent disagreement on the ranges of validity, the leading low-velocity expansions for the scattering phase shifts for the matrix model and supergravity are known to give precisely the same results at long-distance regime $\ell \gg \ell_P$ in lower order perturbation theory with respect to g_s . The supersymmetric nonrenormalization theorem [7] is responsible for this result at least to the first nontrivial order. It is not clear whether the supersymmetry is sufficient to explain a much more nontrivial result [8] at two-loop order which contains the nonlinear self-interaction of graviton. An important task in Matrix theory is to clarify to what extent the coincidence between the matrix model for finite N and the supergravity in the DLCQ limit is valid, and, if the coincidence stops at some point, where and how it occurs. This is not, however, the issue on which I would like to focus in the present talk. For a previous review, see *e.g.* [9]. Let us return to the large N limit.

In contrast to the DLCQ interpretation, the Matrix-theory conjecture requires that boost transformation in going to the infinite-momentum frame is equivalent to taking the large N limit with fixed R . Since $P^+ P^- = -N \text{Tr} h/2$ has to be boost-invariant, the whole nontrivial spectrum must then be contained in the region where the spectrum of the operator $\text{Tr} h$ scales as $O(1/N)$. Namely, the Hamiltonian P^- must be scaled as $O(R/N)$. If we study the correlators of the theory instead of the spectrum directly, this amounts to taking the scaling limit with respect to time as $t \rightarrow Nt$. In view of the analogy with light-cone formulation of membranes, it is not unreasonable to suppose that an appropriate large N limit of the matrix model captures the full structure of type IIA/M theory. As we shall discuss shortly, the large N limit indeed enables us to go beyond the DLCQ region, namely, go to the region of length scales much larger than the string scale.

One of the puzzles related to this conjecture is that the holographic property which should be satisfied as a proper quantum theory of gravity requires that the size of the system with respect to the transverse directions must grow indefinitely in the large N boost transformation. The reason is that the boost transformation increases the number of partons. Since the information carried by a single parton is expected to be coded in a transverse volume at most of Planck size, the transverse volume of the total system must grow at least as fast as N which means that the transverse size is at least of order

$O(N^{1/9})$ in the limit $N \rightarrow \infty$. Thus a crucial issue in studying the large N behavior of Matrix theory is how to reconcile these properties with the boost invariance. In the light-cone formulation of string theory, we parametrize the strings such that the density of the light-like momentum is uniform along the string and also is constant with respect to the light-cone time. The number of string bits is then proportional to the light-like momentum. However no violation of Lorentz invariance occurs in the final results of scattering amplitudes. In Matrix theory, similarly, there must be some mechanism by which the apparent growth of the transverse size could be compatible with 11 dimensional Lorentz invariance .

2. Generalized conformal symmetry and oblique AdS/CFT correspondence

Let us now consider the following question: Can we extract any nontrivial information on the large N limit of Matrix theory from the AdS/CFT type correspondence? This problem must have been touched upon by many authors from different perspectives. Due to the limited space, I can only mention a few [10] [11] which are closely related to the present talk. For other related works, I recommend the readers to consult references in these cited works. However, to my knowledge, no concrete results have been reported on the behavior of the correlation functions of Matrix theory along this direction. In our previous works [2, 3], we have proposed to approach the problem from the viewpoint of a generalized conformal symmetry.

In the usual AdS/CFT correspondence, the existence of conformal symmetry plays crucial roles. The classification of the spectrum with respect to the conformal symmetry on both sides of bulk and boundary theory and agreement between them are the strongest piece of evidence for the correspondence. Furthermore, the conjectured correspondence of the correlators between the bulk theory and the conformal field theory at the boundary of the near-horizon region again relies upon the conformal symmetry of the bulk theory in the whole near-horizon region. Therefore it is natural to seek the possible generalization of the conformal symmetry for non-conformal branes in extending the correspondence to non-conformal D-branes.

Let us start by examining the structure of the classical D0-solution:

$$ds_{10}^2 = -e^{-2\tilde{\phi}/3} dt^2 + e^{2\tilde{\phi}/3} dx_i^2, \quad e^\phi = g_s e^{\tilde{\phi}} \quad (2.5)$$

$$e^{\tilde{\phi}} = \left(1 + \frac{q}{r^7}\right)^{3/4}, \quad A_0 = -\frac{1}{g_s} \left(\frac{1}{1 + \frac{q}{r^7}} - 1\right), \quad (2.6)$$

where the charge q is given by $q = 60\pi^3(\alpha')^{7/2}g_s N$. In the near horizon limit $q/r^7 \gg 1$, the factor $1 + q/r^7$ is replaced by q/r^7 and the metric is rewritten as

$$ds_{10}^2 = -\frac{r^2}{\rho^2} dt^2 + \frac{\rho^2}{r^2} (dr^2 + r^2 d\Omega_8^2), \quad \rho = \rho(r) = \left(\frac{q}{r^3}\right)^{1/4}. \quad (2.7)$$

We can check that the metric, dilaton and the 1-form $A_0 dt$ are all invariant under the scale and the special conformal transformations if they are accompanied by the

transformations of the string coupling as

$$r \rightarrow \lambda r, t \rightarrow \lambda^{-1} t, g_s \rightarrow \lambda^3 g_s, \quad (2.8)$$

$$\delta_K t = -\epsilon(t^2 + \frac{2q}{5r^5}), \quad \delta_K r = 2\epsilon t r, \quad \delta_K g_s = 6\epsilon t g_s, \quad (2.9)$$

which together with time translation form an $SO(1,2)$ algebra. An important feature of this generalized conformal symmetry is that the would-be AdS radius ρ as a function of r is invariant under the transformation. Namely, although the background space-time is *not* AdS \times S⁸, it behaves almost like that, since the r -dependent radius $\rho(r)$ is invariant.

Furthermore, the same generalized symmetry is satisfied for Matrix-theory lagrangian:

$$X_i \rightarrow \lambda X_i, t \rightarrow \lambda^{-1} t, g_s \rightarrow \lambda^3 g_s, \quad (2.10)$$

$$\delta_K X_i = 2\epsilon t X_i, \delta_K t = -\epsilon t^2, \delta_K g_s = 6\epsilon t g_s. \quad (2.11)$$

The difference between $\delta_K t$ in (2.9) and (2.11) has the same origin as in the usual case of D3-brane: The mechanism how the additional term $\frac{2q}{5r^5}$ in (2.11) emerges in the bulk theory was clarified in refs. [3] for general case of Dp -branes from the point of view of matrix models, namely, from the boundary theory. I also remark that this generalization of conformal symmetry has been motivated at a deeper level by the space-time uncertainty principle [12] as has been discussed in [2] in detail.

Another comment which is perhaps worthwhile to make here is that the generalized conformal symmetry is regarded as the underlying symmetry for the DLCQ interpretation. We are free to change the engineering scales. Thus, if one wants to keep the numerical value of the transverse dimensions, we perform a rescaling $t \rightarrow \lambda^{-1} t, X_i \rightarrow \lambda^{-1} X_i, \ell_s \rightarrow \lambda^{-1} \ell_s$ simultaneously with the generalized scaling transformation leading to the scaling $t \rightarrow \lambda^{-2} t, X_i \rightarrow X_i, R \rightarrow \lambda^2 R$ and $\ell_P \rightarrow \ell_P$ which is equivalent with the kinematical boost transformation. Alternatively, one might want to keep the numerical value of time or energy by making a rescaling $t \rightarrow \lambda t, X_i \rightarrow \lambda X_i, \ell_s \rightarrow \lambda \ell_s$, leading to the scaling $t \rightarrow t, R \rightarrow \lambda^4 R, X_i \rightarrow \lambda^2 X_i, \ell_P \rightarrow \lambda^2 \ell_P$ and $\ell_s \rightarrow \lambda \ell_s$, which is in fact equivalent to the ‘tilde’ transformation utilized in [6]. Note that although the second case makes the string length ℓ_s small by assuming small λ , the length scale of transverse directions smaller than the string scale is always sent to even smaller length scale ($< \lambda^2 \ell_s$).

In view of these symmetry properties, it is quite natural to suppose that the similar correspondence between supergravity and Yang-Mills matrix quantum mechanics is valid as in the typical case of D3-brane between supergravity and 4 dimensional super Yang-Mills theory. We now examine the conditions [13] [14] for the validity of the correspondence. For comparison, we indicate the corresponding conditions for the case of D3-brane in parentheses. We neglect numerical coefficients in writing down these conditions.

- Near horizon condition :

$$r \ll \rho(r) \rightarrow r \ll (g_s N)^{1/7} \ell_s, \quad (r \ll (g_s N)^{1/4} \ell_s : \text{D3}) \quad (2.12)$$

- Small curvature condition :

$$\rho(r) \gg \ell_s \rightarrow r \ll (g_s N)^{1/3} \ell_s, \quad ((g_s N)^{1/4} \ell_s \gg \ell_s : \text{D3}) \quad (2.13)$$

- Small string-coupling condition:

$$g_s e^{\tilde{\phi}} = e^{\phi} \ll 1 \rightarrow (g_s N)^{1/3} N^{-4/21} \ell_s \ll r. \quad (g_s \ll 1 : \text{D3}) \quad (2.14)$$

In both cases of D0 and D3, the near horizon conditions (2.12) are *by definition* not invariant under the (generalized) conformal transformation, while the other two conditions (2.13) and (2.14) are invariant. The former indicates that the boundary of the near-horizon or conformal region should be assumed at $r \sim q^{1/7}$ ($r \sim (g_s N)^{1/4} : \text{D3}$). Note that in the case of ordinary AdS/CFT correspondence, both the near-horizon condition and the small curvature conditions are characterized by a single scale $(g_s N)^{1/4} \ell_s = \ell_{10} N^{1/4}$, while in the present case they are different.

Both the difference and the common features in the above conditions for D0 and D3 are best illustrated by Figures 1 and 2 below. In Fig. 1, the lines representing the

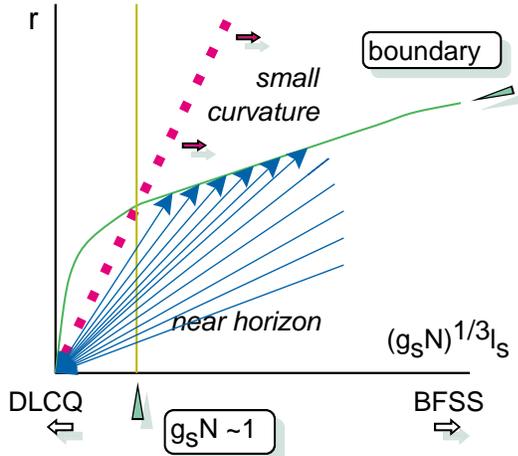


Fig.1 : Oblique AdS/CFT
correspondence

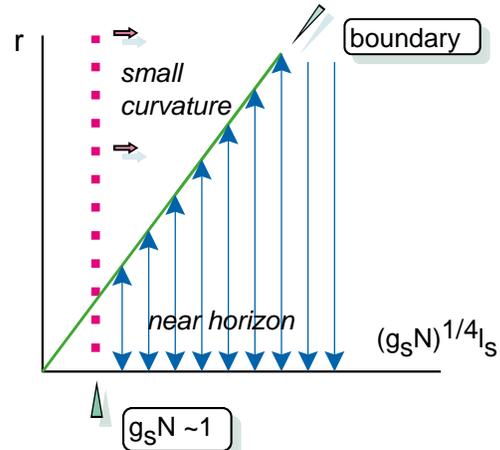


Fig 2 : Ordinary AdS/CFT
correspondence

near-horizon boundary and the small-curvature boundary (dashed), as well as the flow lines (with arrows) of the generalized conformal transformation, are tilted comparing to the corresponding lines of Fig. 2, while the topologies of those lines are essentially the same for $r > 0$. Note that the condition of small string coupling is automatically satisfied for finite $r > 0$ in the 't Hooft limit by keeping $g_s N$ fixed. In the oblique case, contrary to the ordinary case, the near horizon region with small effective curvature contains the DLCQ region $g_s N < 1$. This is due to the r dependence of the quasi AdS size $\rho(r)$. However, this does not mean that the correspondence is valid when restricted in the weak coupling region, since by the generalized conformal transformation even the DLCQ region is sent to the strong coupling region $g_s N > 1$ at the near-horizon boundary.

On the other hand, the limit of large coupling 't Hooft $g_s N \rightarrow \infty$ enables us to approach the BFSS region. Here, it is crucial to realize that the oblique AdS/CFT correspondence is limited at the boundary of the near horizon condition which is at the distance of order $r \sim q^{1/7} \sim (g_s N)^{1/7} \ell_s$, just like that the ordinary AdS/CFT correspondence for D3 case is limited at the boundary $r \sim (g_s N)^{1/4} \ell_s$. It is natural to suppose that these distances are the infrared cutoffs for the corresponding Yang-Mills theory with respect to the range of values of transverse coordinates, namely, the cutoffs for the magnitude of the Higgs fields X_i .

3. Predictions for 2-point functions and anomalous large N scaling behavior

Armed by these considerations, we are now ready to discuss the large N behavior of the two-point correlators. The strategy for doing this is the following. First step: We establish the correspondence between the supergravity fluctuations and the matrix-model operators relying upon the generalized conformal symmetry. Namely, we derive the two-point correlators assuming the now familiar conjecture that the supergravity action is the generating functional of the matrix model correlators. We will follow the prescription of [15], which assumes the boundary at $r \sim q^{1/7}$ ($r \sim (g_s N)^{1/4} \ell_s$: D3) and extracts the universal part of the correlators as the nonanalytically (in momentum space) behaved part. This is more convenient for us than the more formal prescription adopted in [16]. We also note that the singularity of the metric and the dilaton essentially cancels at the horizon and the behavior of the kinetic radial term of the effective action at the horizon is not much different from the D3 case: *e. g.* $\partial_\mu \sqrt{g} e^{-2\phi} g^{\mu\nu} \partial_\nu \propto r^8 (\partial_r^2 + 8 \frac{1}{r} \partial_r - \frac{q}{r^7} \partial_t^2)$ as $r \rightarrow 0$ for the scalar field. Hence the singularity does not cause any harm for our program. As remarked previously, the generalized scaling transformation is equivalent to the kinematical boost transformation. Thus the first step amounts to utilizing the DLCQ interpretation of the matrix model. Since the range of validity for the oblique AdS/CFT correspondence contains the DLCQ region ($g_s N \rightarrow 0$) and BFSS region ($g_s N \rightarrow \infty$) as two opposite extreme limits (see Fig. 1), we can expect that the behavior of the correlators should also be consistent with the BFSS interpretation, if the latter is indeed correct. The second step is then to consider the large N scaling behavior of the correlators by making the rescaling $t \rightarrow Nt$.

Let us now determine the general form of the two-point correlators using the relation

$$e^{-S_{\text{Sugra}}[\phi_0]} = \langle e^{\int d\tau \sum_I \phi_0^I \mathcal{O}_I} \rangle, \quad \phi^I(x)|_{r=r_b} = \phi_0^I, \quad r_b = q^{1/7} \quad (3.15)$$

where τ is of course the Wick-rotated euclidian time and I labels independent *diagonalized* degrees of freedom in the spectrum of the supergravity fluctuations and the corresponding matrix-model operators with definite generalized conformal dimensions. This shows that the connected two-point functions are just given by the supergravity action evaluated to the second order of the boundary value ϕ_0^I . Since, apart from the overall 10 dimensional Newton constant $g_s^2 \ell_s^8$, variable constants (g_s, N, ℓ_s) enter in the effective action only through $q \sim g_s N \ell_s^7$, the generalized conformal symmetry is

sufficient to fix the general form of the two-point functions as

$$\langle \mathcal{O}_I(\tau_1) \mathcal{O}_I(\tau_2) \rangle_c \sim \frac{1}{g_s^2 \ell_s^8} q^{(\Delta_I+6)/5} |\tau_1 - \tau_2|^{-(7\Delta_I+12)/5} \quad (3.16)$$

where we have assumed that the matrix-model operators are normalized such that their engineering dimensions with respect to the length is -1 , and Δ_I is the generalized conformal dimension of the operator; namely $\mathcal{O}_I(\tau) \rightarrow \mathcal{O}'_I(\tau') = \lambda^{\Delta_I} \mathcal{O}_I(\tau)$, $\tau' = \lambda^{-1} \tau$, $g_s \rightarrow g'_s = \lambda^3 g_s$ for scaling, and similarly for special conformal transformation.

In [4], we have performed a complete harmonic analysis of the bosonic fluctuations around the D0 background and explicitly confirmed the validity of the above formula for all the bosonic physical fluctuations of supergravity. The bosonic fluctuations are described by the modified Bessel equation, $(-\partial_z^2 - \frac{1}{z} \partial_z - \partial_\tau^2 + \frac{\nu_I^2}{z^2}) \phi^I(z, \tau) = 0$, where $z = 2q^{1/2} r^{-5/2}/5$ is the ‘quasi’-Poincaré coordinate and the order of the Bessel function is related to the generalized conformal dimension by $\Delta_I = -1 + \frac{10}{7} \nu_I$. The spectrum of the dimensions Δ_I is classified from the 11 dimensional viewpoint as $\Delta_I = -1 + 2n_I + \frac{4}{7} \ell_I$ where $n_I = 1 - n_+ + n_-$ is determined by the kinematical boost dimensions n_\pm , which are nothing but the number of upper light-cone indices \pm respectively, and ℓ_I is the order of the harmonics. The fractional dependence on the order of the harmonics comes from our normalization of the operators such that their engineering dimensions are uniformly -1 , which leads to the harmonic expansion in terms of the normalized transverse coordinates $\tilde{X}_i = X_i/q^{1/7}$. The results for the generalized conformal dimensions are consistent with the known results [18] from the lowest order perturbative computation for the matrix-model operators coupled with supergravity fields. Note that since we are just dealing with quantum mechanics there is no problem in determining the (generalized) conformal dimensions of these operators. A few examples of the operators $\mathcal{O}(\tau)$ are

$$\begin{aligned} \Delta = -3 + \frac{4\ell}{7} : T_{\ell, i_1 i_2 \dots i_\ell}^{++} &= \frac{1}{R} \text{STr}(\tilde{X}_{i_1} \tilde{X}_{i_2} \dots \tilde{X}_{i_\ell} + \dots), \quad (\ell \geq 2) \\ \Delta = -1 + \frac{4\ell}{7} : T_{\ell, i_1 i_2 \dots i_\ell}^{+i} &= \frac{1}{R} \text{STr}(\dot{X}_i \tilde{X}_{i_1} \tilde{X}_{i_2} \dots \tilde{X}_{i_\ell} + \dots), \quad (\ell \geq 2) \\ \Delta = +1 + \frac{4\ell}{7} : T_{\ell, i_1 i_2 \dots i_\ell}^{ij} &= \frac{1}{R} \text{STr}(\dot{X}_i \dot{X}_j \tilde{X}_{i_1} \tilde{X}_{i_2} \dots \tilde{X}_{i_\ell} + \dots), \quad (\ell \geq 2) \end{aligned}$$

etc.

For more details, I would like to invite the reader to the paper [4].

Let me mention some notable features of our results before discussing the implications for the BFSS conjecture. First, the correlators have fractional dependence on both g_s and N , which can never be reproduced from the perturbative computations of the correlators. This is not surprising if we recall that such perturbative computations would be plagued by infrared divergencies. Furthermore, the angular-momentum independent part of the g_s -and N -dependencies of the the dilaton-10D energy-momentum correlators ($\Delta = 1 + \frac{4}{7} \ell$) agree with that of the entropy [19] of the nonextremal D0 solution at a given temperature T_H : $S \sim N^2 (g_s N)^{-3/5} (\ell_s T_H)^{9/5}$. This provides evidence for the fact that the correlator corresponding to the energy-momentum tensor without mixing of other modes adequately counts the number of degrees of

freedom in the low-energy regime of many D-particle dynamics from 10 dimensional viewpoint.

Let us finally discuss the large N scaling behavior of the result (3.16) and its implications. By making the scaling for the time $\tau_1 - \tau_2 \rightarrow N(\tau_1 - \tau_2)$, we find that the correlators scale as $N^{2d_{IMF}}$ with

$$d_{IMF} = \left(1 + \frac{1}{5}\right)(n_+ - n_- - 1) - \left(\frac{1}{5} + \frac{1}{7}\right)\ell. \quad (3.17)$$

It is remarkable that except for the additional fraction $1/5$ in both the first and the second term in (3.17), this is just consistent with the boost transformation of the operators. Note that the fraction $-1/7$ just accounts for one factor of N in $1/q^{1/7}$ coming from the normalization. What is not clear is the origin of the anomalous dimensions $-1/5$ for the transverse directions and $\pm 1/5$ for the upper light-cone indices \pm . In particular, the large N scaling implies a *shrinking* behavior $N^{-1/5}$ in the transverse direction, which is quite opposite to what we naively expect from holography!

Does this contradict holography? Not necessarily. The reason is that the large-time correlators of the operators with higher partial waves are not directly measuring the extension of wave functions. It is conceivable that the complicated time-dependent dynamics effectively screens the correlation with respect to the transverse extension of the wave functions. After all, it is very hard to believe that the dynamics in the large N IMF could be consistent with supergravity, unless the holographic growth of the system is somehow screened to become an unobservable effect. Here we have to recall that the oblique AdS/CFT correspondence on which the above results are based has an intrinsic infrared cutoff $r < r_B \sim q^{1/7} \propto N^{1/7}$. This bound is bigger than the well known mean-field estimate $N^{1/9}$. The same estimate is also obtained from a simple counting of the degree of freedom for describing the Schwarzschild black hole in Matrix theory [17]. However, it is much smaller than the lower bound of order $N^{1/3}$ [10] for the eigenvalue distribution which is derived using a virial theorem. Thus, the oblique AdS/CFT correspondence only enables us to predict the large N behaviors of the system put in a box whose size is much smaller than the real quantum system, while it is bigger than the classical size of the individual objects. To discuss the system whose size is consistent with the lower bound, we have to renormalize the system to bigger sizes. This is a difficult dynamical problem.

Here we may reverse the direction of the arguments. Instead of directly studying the size renormalization, we can ask what the size of the system must be if we demand that the large N behavior be consistent with boost invariance. Since the consistency with boost invariance requires that there should be no anomalous dimensions with respect to the transverse directions, let us change the cutoff for the distribution of the eigenvalue by making a scaling $X_i \rightarrow N^{1/5} X_i$ at *fixed* g_s and N . If we assume that the similarity law is satisfied for this system, the scaling changes the infrared cutoff from the order $N^{1/7}$ to the order $N^{1/5} \times N^{1/7} = N^{(1/3)+\epsilon}$, $\epsilon = 1/105$. This is consistent with and is very close to the lower bound $\propto N^{1/3}$. Although this argument is very naive and it is not at all clear whether this procedure eliminates the anomalous dimensions in the

light-like directions too,§ it might be regarded as a signal for the consistency of the large N scaling behavior with 11 dimensional boost invariance. We should however keep in mind also that our discussion is limited in the region of weak string coupling no matter how $g_s N$ is large.

Apart from clarifying the above question, there are many future problems which should be studied. To conclude, I enumerate a few of them: (1) the study of the representation theory of the super generalized conformal algebra, (2) the computations of three and higher-point functions and their implications, and (3) possible applications to the computation of S-matrix in the large N limit.

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§ The anomalous dimensions in the light-like direction might be eliminated by simply assuming that the dynamical boost should be accompanied with a kinematical boost.