Baryons and Flux Tubes in Confining Gauge Theories from Brane Actions

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Abstract

We study baryon configurations in large N non-supersymmetric $SU(N)$ gauge theories, applying the AdS/CFT correspondence. Using the D5-brane worldvolume theory in the near-horizon geometry of non-extremal D3-branes, we find embeddings which describe baryonic states in three-dimensional QCD. In particular, we construct solutions corresponding to a baryon made of N quarks, and study what happens when some fraction ν of the total number of quarks are bodily moved to a large spatial separation from the others. The individual clumps of quarks are represented by Born-Infeld string tubes obtained from a D5-brane whose spatial section has topology $\mathbf{R} \times \mathbf{S}^4$. They are connected by a confining color flux tube, described by a portion of the fivebrane that runs very close and parallel to the horizon. We find that this flux tube has a tension with a nontrivial ν-dependence (not previously obtained by other methods). A similar picture is presented for the four-dimensional case.

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1 Introduction

The Born-Infeld action plus an appropriate Wess-Zumino term defines a worldvolume theory for D-branes which has proved to be a powerful way to describe these objects and their excitations. In the context of Maldacena's correspondence between supergravity in anti-de Sitter (AdS) space and certain gauge theories [1, 2, 3], there are gauge theory questions which can be answered by the study of branes extended in curved space. In particular, it was shown in a general way that the large-N_c dynamics of baryons can be related to the behavior of D5-branes extended in AdS space [4, 5]. Concrete realizations of this possiblity in the context of the non-confining $\mathcal{N} = 4$ supersymmetric gauge theory have been worked out in [6, 7] using the Born-Infeld approach for constructing strings out of D-branes [8, 9]. In this paper, we will look at how these constructions work in the more complicated spacetimes that correspond to various confining gauge theories. We will examine confining forces by looking at what happens to baryons when they are pulled apart into their quark constituents. This will be compared to (and yields somewhat more information than) the study of confining forces via simple strings that 'hang' into the AdS geometry from a boundary Wilson loop [10, 11, 5].

In this paper, we extend our previous work [6] on D5-branes in extremal background in three ways. First, we will study non-extremal supergravity backgrounds, corresponding to gauge theories dimensionally reduced in a way that breaks supersymmetry. Second, we allow the brane configurations to have extension in the spacetime coordinates of the gauge-theory instead of being localized at a point. This will allow us to describe a baryon which is being 'pulled apart' into quark constituents. Third, we discuss also the case based on D4-branes, which corresponds to a gauge theory in four dimensions.

We start out in Section 2 by analysing baryons in a (2+1)-dimensional $SU(N)$ Yang-Mills theory which is obtained from (3+1)-dimensions by compactifying on a supersymmetry breaking circle. As proposed in [12], this gauge theory is dual to a certain non-extremal D3-brane geometry[5] and, following [6], we study solutions of the D5-brane worldvolume equations of motion in that geometry. We find a class of solutions that are localized in the gauge theory spatial coordinates and appear to describe the baryon. Unlike the baryons constructed in the extremal background [6, 7], these solutions have no moduli since the quarks are truly bound in the baryon. We then study a new class of solutions in which the N quarks are separated into two groups, containing νN and $(1 - \nu)N$ quarks respectively, separated by a spatial distance L in the gauge theory. The L-dependence of the energy of these solutions is consistent with confine-

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[5]See [13, 14, 15] for some interesting alternative string theory approaches to the study of large-N non-supersymmetric Yang-Mills theories.

1

ment and the implied color flux tube tension has a non-trivial dependence on the color charge ν.

In Section 3 we carry out the same analysis for the gauge theory in one more dimension, i.e., D4-branes embedded in a non-extremal D4-brane geometry. We include also a short section on the extremal limit, as this was not covered in [6]. The analysis is parallel to the one in Section 2, with the added surprise that the resulting color flux tension now has a very simple dependence on ν.

In a final section we summarize our results and discuss future directions.

2 The Baryon in Three Dimensions

2.1 Worldvolume Action and Equations of Motion

We derive the equations for a D5-brane embedded in the near-horizon geometry of N nonextremal D3-branes. The Euclidean background metric is

$$ds^2 = \left(\frac{r}{R}\right)^2 \left[f(r)d\tau^2 + dx_{||}^2\right] + \left(\frac{R}{r}\right)^2 f(r)^{-1}dr^2 + R^2 d\Omega_5^2, \tag{1}$$

$$f(r) = 1 - r_h^4/r^4, \qquad R^4 = 4\pi g_s N l_s{}^4, \qquad r_h = \pi R^2 T,$$

where $\{\tau, x_{||}\} = \{\tau, x, y, z\}$ denote the directions parallel to the threebranes. The coordinate τ is periodic, with period $1/T$, where T is the Hawking temperature. The relation between the horizon radius r_h and T ensures smoothness of the geometry at $r = r_h$.

Under the AdS/CFT correspondence [1, 2, 3], type IIB string theory on a background with the above metric, a constant dilaton, and N units of fiveform flux through the five-sphere, is dual to $\mathcal{N} = 4$, $d = 3 + 1$ $SU(N)$ SYM theory at temperature T, with coupling $g_{YM_4}^2 = 2\pi g_s$. The gauge theory coordinates are $\{x, y, z, \tau\}$. For large T the τ circle becomes small and one effectively obtains a description of a strongly-coupled (coupling $g_{YM_3}^2 = g_{YM_4}^2 T$) three-dimensional Euclidean gauge theory at zero temperature. The thermal boundary conditions on the circle break supersymmetry and the fermions and scalars acquire masses of order T and $g_{YM_4}^2 T$, respectively. The effective three-dimensional theory is expected to display behavior similar to that of non-supersymmetric pure Yang-Mills theory, QCD_3 [12].

A baryon (a bound state of N external quarks) in the three-dimensional theory has as its string theory counterpart a fivebrane wrapped on an $\mathbf{S}^5$ on which N fundamental strings terminate [4, 5]. The fivebrane worldvolume action is

$$S = -T_5 \int d^6\xi \sqrt{\det(g + F)} + T_5 \int A_{(1)} \wedge G_{(5)} \, ,$$

2

where $T_5 = 1/(g_s(2\pi)^5 l_s^{\,6})$ is the brane tension. The Born-Infeld term involves the induced metric g and the $U(1)$ worldvolume field strength $F_{(2)} = dA_{(1)}$. The second term is the Wess-Zumino coupling of the worldvolume gauge field $A_{(1)}$ to (the pullback of) the background five-form field strength $G_{(5)}$, which effectively endows the fivebrane with a $U(1)$ charge proportional to the $\mathbf{S}^5$ solid angle that it spans.

For a static baryon we need a configuration invariant under translations in the gauge theory time direction, which we take to be y. We use y and the $\mathbf{S}^5$ spherical coordinates as worldvolume coordinates for the fivebrane, $\xi_\alpha = (y, \theta_\alpha)$. For simplicity we restrict our attention to $SO(5)$ symmetric configurations of the form $r(\theta)$, $x(\theta)$, and $A_y(\theta)$ (with all other fields set to zero), where θ is the polar angle in spherical coordinates. The action then simplifies to

$$S = T_5 \Omega_4 R^4 \int dy \, d\theta \, \sin^4 \theta \{ -\sqrt{r^2 + r'^2/f(r) + (r/R)^4 x'^2 - F_{y\theta}^2} + 4A_y \}, \qquad (2)$$

where $\Omega_4 = 8\pi^2/3$ is the volume of the unit four-sphere.

The gauge field equation of motion following from this action reads

$$\partial_\theta D(\theta) = -4 \sin^4 \theta,$$

where the dimensionless displacement D is the variation of the action with respect to $E = F_{y\theta}$. The solution to this equation is

$$D(\theta) = \left[\frac{3}{2}(\nu\pi - \theta) + \frac{3}{2} \sin\theta \cos\theta + \sin^3\theta \cos\theta \right]. \qquad (3)$$

As will be explained below, the integration constant $0 \le \nu \le 1$ controls the number of Born-Infeld strings emerging from each pole of the $\mathbf{S}^5$. Next, it is convenient to eliminate the gauge field in favor of D and Legendre transform the original Lagrangian to obtain an energy functional of the embedding coordinate $r(\theta)$ only:

$$U = T_5 \Omega_4 R^4 \int d\theta \sqrt{r^2 + r'^2/f(r) + (r/R)^4 x'^2} \sqrt{D(\theta)^2 + \sin^8 \theta} \,. \qquad (4)$$

This action has the interesting scaling property that if $\{r(\theta), x(\theta)\}$ is a solution for horizon radius r_h, then $\{\alpha r(\theta), \alpha^{-1} x(\theta)\}$ is a solution for horizon radius αr_h. The scaling $x \propto R^2/r$ is precisely as expected from the holographic UV/IR relation [16, 17]. We will have more to say about scaling behavior of solutions later on.

The fivebrane embeddings of interest to us will have singularities: places on the five-sphere (typically $\theta \to \pi$ or 0) where $r \to \infty$ and $x' \to 0$. As in [6, 8, 9], these 'spikes' must be interpreted as bundles of fundamental strings attached to the wrapped fivebrane and localized at some definite value of x. It can be seen from (4) that a spike sticking out at $\theta = \pi$ has a 'tension' (energy per unit radial coordinate distance)

3

$T_5 \Omega_4 R^4 |D(\pi)| f(r)^{-1/2} = (1-\nu) N T_F f(r)^{-1/2}$, which is precisely the tension of $(1-\nu)N$ fundamental strings in this geometry. A spike at $\theta = 0$ has the same tension as νN strings so that, taken together, the two singularities represent a total of N fundamental strings, as expected. Surfaces with more singularities and less symmetry are perfectly possible, but a lot harder to analyze. To keep things manageable, we have built $SO(5)$ symmetry into the ansatz.

In the extremal case ($r_h = 0$) discussed in [6], the BPS condition provided a first integral which greatly simplified the analysis. In the nonextremal case we are now discussing, there is no such first integral and we have to deal with the unpleasant second order Euler-Lagrange equation that follows from (4). This is most conveniently done in a parametric Hamiltonian formalism [6]. First we rewrite the energy in terms of a general worldvolume parameter s with the D5-brane embedding defined by functions $\theta = \theta(s)$, $r = r(s)$, $x = x(s)$:

$$U = T_5 \Omega_4 R^4 \int ds \sqrt{r^2 \dot{\theta}^2 + \dot{r}^2/f + (r/R)^4 \dot{x}^2} \sqrt{D^2 + \sin^8 \theta}, \tag{5}$$

where dots denote derivatives with respect to s. The momenta conjugate to r, θ and x are

$$p_r = f^{-1} \dot{r} \Delta, \quad p_\theta = r^2 \dot{\theta} \Delta, \quad p_x = (r/R)^4 \dot{x} \Delta, \quad \Delta = \frac{\sqrt{D^2 + \sin^8 \theta}}{\sqrt{r^2 \dot{\theta}^2 + \dot{r}^2/f + (r/R)^4 \dot{x}^2}}. \tag{6}$$

The Hamiltonian that follows from the action (5) vanishes identically due to reparametrization invariance in s. Furthermore, the momentum expressions are non-invertible and the system is subject to the constraint

$$2\tilde{H} = \left(1 - \frac{r_h^4}{r^4}\right) p_r^2 + \frac{p_\theta^2}{r^2} + \frac{R^4}{r^4} p_x^2 - \left(D^2 + \sin^8 \theta\right) = 0 . \tag{7}$$

This constraint can be taken as the Hamiltonian and this choice conveniently fixes the gauge, while getting rid of the complicated square-root structure of the action. The equations of motion that follow from this Hamiltonian are

$$
\begin{aligned}
\dot{r} &= \left(1 - \frac{r_h^4}{r^4}\right) p_r , & \dot{p}_r &= \frac{2}{r^5}(p_x^2 R^4 - p_r^2 r_h^4) + \frac{p_\theta^2}{r^3} , \\
\dot{\theta} &= \frac{p_\theta}{r^2} , & \dot{p}_\theta &= -6 \sin^4 \theta \left(\pi\nu - \theta + \sin\theta\cos\theta\right) , \\
\dot{x} &= \frac{R^4}{r^4} p_x, & \dot{p}_x &= 0 .
\end{aligned}
\tag{8}
$$

Together with initial conditions, these equations completely define the solutions for the fivebrane. The initial conditions should be chosen such that $\tilde{H} = 0$.

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[6] We would like to thank G. Savvidy for suggesting and helping to realize this approach.

To gain some insight into the solutions to these equations, notice that the basic problem to solve is a motion in the two-dimensional $r - \theta$ plane: the motion in x is then determined by the choice of a conserved value for p_x. Note that for $p_x = 0$, the surface sits at a fixed value of x and therefore has no spatial extension in the gauge theory coordinates: we will call this a 'point' solution. When r is large compared to r_h and $R\sqrt{p_x}$, the $r - \theta$ motion is simply that of a particle of unit mass moving in two dimensions under the influence of the angular potential $V(\theta) = -\left[D(\theta)^2 + \sin^8 \theta\right]$. By the constraint, the energy of this fictitious particle vanishes. For generic ν, the potential has three extrema (see Figure 1): two minima at $\theta = 0$ and $\theta = \pi$, and a maximum at $\theta = \theta_c$ such that $\pi\nu = \theta_c - \sin \theta_c \cos \theta_c$. For large r the particle will thus roll down towards one of the two minima. Whether it reaches $\theta = 0, \pi$ at a finite value of r depends on the initial boundary conditions.
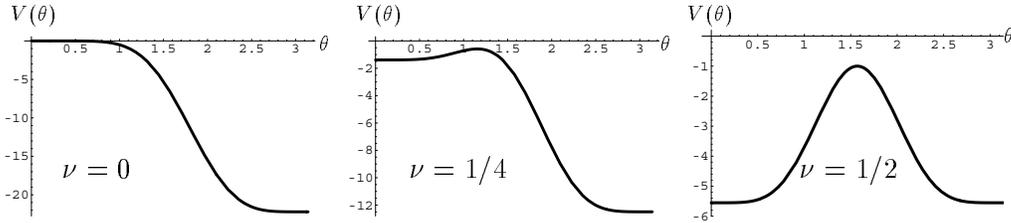

Figure 1: The potential $V(\theta) = -\left[D(\theta)^2 + \sin^8 \theta\right]$ for $\nu = 0, 1/4, 1/2$ (see text for discussion).

2.2 The Point Baryon

In this section we study solutions which correspond to a baryon localized at a particular gauge theory position. To localize the surface in x, we just set $p_x = 0$. With the symmetry that we have built in, the equations of motion typically allow the surface to run off to $r = \infty$ at $\theta = \pi$ or 0. At least asymptotically, such 'spikes' are equivalent to bundles of fundamental strings and will be identified with the quark constituents of the state represented by the wrapped fivebrane. To get a baryon whose component quarks have identical $SU(4)$ (flavor) quantum numbers, we want a spike representing N quarks to emerge from one pole of the S^5 (say $\theta = \pi$) with a smooth surface at the other pole. To meet the first condition, it suffices to set the integration constant $\nu = 0$ and to meet the second, we impose smooth boundary conditions ($\partial_\theta r = 0$ and $r = r_0$) at $\theta = 0$.[7]

Numerical integration with these boundary conditions yields a one-parameter family of solutions (parametrized by r_0). Due to the scaling mentioned before, the solutions

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[7]This is equivalent to requiring $p_r = 0$ at $\theta = 0$, in which case p_θ must also vanish to satisfy the constraint (7).
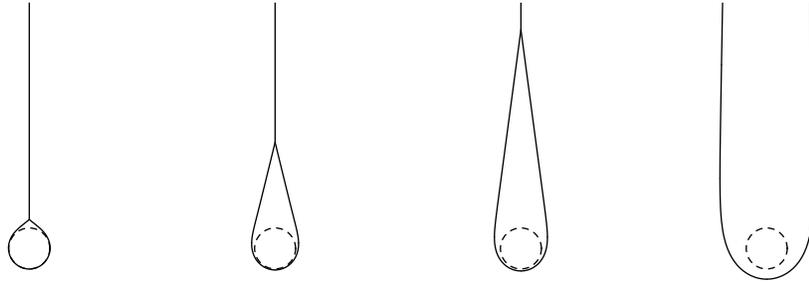
Figure 2: Family of solutions illustrating the progressive deformation of the fivebrane by the bundle of N fundamental strings. The dotted circle represents the horizon. The stable configuration is a 'tube' with $r_0 \to \infty$.

really only depend on r_h through the ratio $\zeta = r_0/r_h \geq 1$. Figure 2 shows polar plots[8] of the solutions for a few representative values of ζ. When $\zeta < \zeta_{crit} \approx 9$ the solution tilts toward and eventually crosses the symmetry axis, thus reaching $\theta = \pi$ at a finite value of r. As $\zeta \to 1$ (i.e. as the starting radius gets closer and closer to the horizon), the brane becomes more and more spherical (notice that $r(\theta) = r_h$ is a solution to the equations of motion). For $\zeta > \zeta_{crit}$, on the other hand, the solution tilts away from the symmetry axis, reaching $\theta = \pi$ only at $r = \infty$. As ζ increases the solution looks more and more like a 'tube'. This is simply a consequence of the fact that, for large ζ, the solution is always very far from the horizon, and is well-approximated by an extremal embedding (discussed in [6]). When $\zeta \to \infty$ the configuration becomes a BPS 'tube' [6] with infinite radius.

The cusp in the $\zeta < \zeta_{crit}$ solutions indicates the presence of a delta-function source in the equations of motion. Since the compactly wrapped brane intercepts N units of five-form flux, it has N units of worldbrane $U(1)$ charge and must have N fundamental strings attached to it [4]. This is most simply achieved by taking the cusp as the point of attachment of N fundamental strings, running along the ray $\theta = \pi$. In accordance with the Born-Infeld string philosophy [8, 9], these strings are equivalent to a D5-brane wrapped on an S^4 of vanishing volume which carries the $U(1)$ flux out to infinity. A simple modification of the flat space argument [18, 8, 9] shows that such a collapsed fivebrane is a solution to the equations of motion and has 'tension' $N T_F f(r)^{-1/2}$ (exactly the tension exerted by a fundamental string of intrinsic tension T_F in this curved space).

The entire fivebrane-string (or collapsed brane) system will be stable only if there is tension balance between its two components. To obtain the stability condition, let r_c

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[8]Although these plots provide a conveniently simple representation of the solutions, the reader should bear in mind that they are a bit misleading as to the intrinsic geometry, since the radius of the $\mathbf{S}^5$ is R, independent of r.

denote the location of the cusp (which is a function of r_0), and parametrize the family of fivebrane embeddings as $r = r(\theta; r_c)$. Under the variation $r_c \to r_c + \delta r_c$, it can be seen from (4), after an integration by parts and application of the Euler-Lagrange equation, that the energy of the brane changes only by a surface term,

$$\frac{\partial U}{\partial r_c} = T_5 \Omega_4 R^4 \frac{r'\sqrt{D^2 + \sin^8\theta}}{f\sqrt{r^2 + f(r)^{-1}r'^2}} \left.\frac{\partial r}{\partial r_c}\right|_0^\pi = \frac{NT_F f(r_c)^{-1/2}}{\sqrt{1 + f(r_c)r_c^2/r_c'^2}} , \tag{9}$$

where $r_c' = \partial_\theta r|_{\theta=\pi}$, and we have used the fact that $r(\pi; r_c) = r_c$. The numerator in the last expression of (9) is the 'tension' at $r = r_c$ of N fundamental strings, so it is clear that the brane has a lower tension for any $r_c > r_h$. The energy is lowered by expanding the fivebrane and shortening the explicit fundamental string. A similar variational calculation applied to the $\zeta > \zeta_{crit}$ configurations (cut off at a large $r = r_{max}$) shows that the BPS 'tube' at infinity is the lowest energy solution. This is consistent with the results of [19, 20], where the baryon was examined using the pure Nambu-Goto action for the fivebrane wrapped on a sphere. We emphasize that the above variational calculation used solutions of the full Born-Infeld (plus Wess-Zumino) action.

Altogether, then, the solutions depicted in Fig. 2 provide a physically satisfying picture of the process through which the N fundamental strings deform the initially spherical fivebrane, pulling it out to infinity. The final configuration has the shape of a 'tube', just like the BPS embeddings found in [7, 6]. In the supersymmetric case, r_0 was a modulus and the energy of the baryon was independent of the overall scale of the solution. In the nonextremal case examined here, however, there appears to be a potential for that modulus which drives the stable solution out to $r_0 \to \infty$.

The dependence of the fivebrane embedding on the S^5 coordinates encodes the flavor structure (i.e., the $SU(4)$ quantum numbers) of the gauge theory state under consideration. As a result of the UV/IR relation, the AdS radial coordinate r is associated with an energy scale in the gauge theory, $E = r/R^2$ [16, 17]. The embedding $r(\theta)$ consequently associates a particular value of θ to each different distance scale, yielding some sort of $SU(4)$ wavefunction for the baryon. The $SO(5)$ symmetry of the embedding translates into the statement that the baryon is a singlet under the corresponding $SU(4)$ subgroup. Finally, the fact that a given surface spans the range $r \geq r_0$ implies that the dual gauge theory configuration has structure on all length scales from zero up to a characteristic size R^2/r_0. Since the energetically preferred configuration has $r_0 \to \infty$, it is in this sense truly pointlike.

2.3 The Split Baryon: Color Dependence of the String Tension

We now turn our attention to solutions with $p_x \neq 0$ (i.e. $x' \neq 0$). They describe collections of quarks at finite separation in the gauge theory position space and are of interest for exploring confinement issues. It turns out to be rather easy to construct a surface describing an $SU(N)$ baryon split into two distinct groups, containing νN and $(1 - \nu)N$ quarks respectively and separated by a distance L in the x direction. In a confining $SU(N)$ gauge theory, two such quark bundles should be connected by a color flux tube and we will study the Born-Infeld representation of this phenomenon. Each group of quarks corresponds as before to a bundle of Born-Infeld strings, realized in our approach as a singular spike or fivebrane 'tube' with topology $\mathbf{R} \times \mathbf{S}^4$. Remember that we have assumed an $SO(5)$-symmetric configuration, which means that the two singularities representing the quarks must be located at opposite poles of the $\mathbf{S}^5$ (we will put them at $\theta = 0$ and $\theta = \pi$) with corresponding implications about the $SU(4)$ flavor structure of the states we are constructing. More general flavor structures are possible, but we will not try to study these more complicated surfaces. For large spatial separation $L = |x(\infty) - x(-\infty)|$, the portion of the fivebrane that interpolates between the two string bundles runs close to the horizon and it is this part of the surface that encodes the confining flux tube of the gauge theory. The surface equations (8) imply that the part of the surface that has large spatial extent must sit at a constant $\theta = \theta_c$ where $\dot{p}_\theta = 0$. More precisely, it has to sit at the solution of

$$\pi\nu = \theta_c - \sin\theta_c \cos\theta_c \ . \tag{10}$$

corresponding to the unstable maximum of the potential $V(\theta)$ discussed at the end of Section 2.1. The critical angle is a monotonic function of ν, such that $\theta_c(0) = 0$ and $\theta_c(\nu) = \pi - \theta_c(1 - \nu)$. The energetics of the part of the fivebrane that encodes the confining flux tube will depend on θ_c, and therefore ν, in a way that we will now examine in some detail.

Unfortunately, we must resort to numerical analysis to construct specific surfaces of this kind. It is convenient to take the point of closest approach to the horizon as the starting point for the numerical integration. We start the integration off with the initial conditions

$$
\begin{aligned}
r(0) &= r_h + \epsilon \ , & p_r(0) &= 0 \ , \\
\theta(0) &= \theta_c \ , & p_\theta(0) &= \eta \ , \\
x(0) &= 0 \ , & p_x(0) &= (r(0)/R)^2 \sqrt{\sin^6\theta_c - (\eta/r(0))^2} \ .
\end{aligned}
\tag{11}
$$

The distance from the horizon at the point of closest approach is controlled by ϵ. For a given ϵ, we have to 'shoot' in η until we get satisfactory behavior of the quark-like

8

singularities at $\theta \to 0$ and $\theta \to \pi$ (see Section 2.2 for details). Indeed, it is natural to require asymptotically BPS behavior in the region of space where supersymmetry is recovered locally (i.e., far from the horizon). Once that is done, ϵ controls the spatial separation of the two separated quark bundles. Figs. 3 and 4 depict a typical fivebrane embedding obtained by numerical integration, for the case $\nu = 0.9$. It can be seen in Fig. 3 that the brane extends in the x direction mostly in its 'flux tube' portion, at $\theta = \theta_c$ and $r \approx r_h$. The Born-Infeld string 'tubes' corresponding to the two groups of quarks lie essentially at a constant value of x.
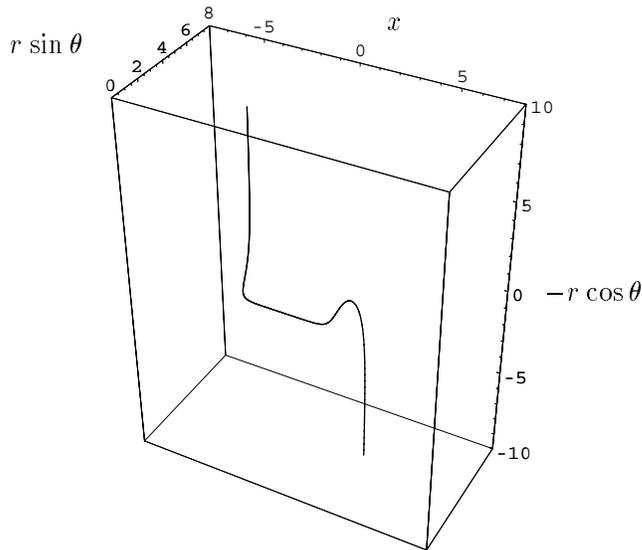

Figure 3: The three-dimensional projection of the D5-brane. Every point on the curve is an $\mathbf{S}^4$. One can clearly see how the brane drops down towards the horizon, extends horizontally along it, and finally leaves at the other end. From the point of view of the three-dimensional $SU(N)$ gauge theory, which lives in the $\{x, y, z\}$ directions, this configuration represents a baryon split into two groups of νN and $(1 - \nu)N$ quarks (the vertical segments — see Fig. 4), connected by a flux tube extending a finite distance along the x direction.

It is seen from the equation for $\dot{r}$ in (8) that, for the portion of the brane running close to the horizon, $r(s) - r_h$ grows as an exponential in the parameter s, with an exponent proportional to p_r. The latter reaches a value close to $(R/r_h)p_x$ (see the equation for $\dot{p}_r$). The equation for $\dot{x}$ then shows that the separation L between the quarks increases only logarithmically with ϵ, the minimal distance to the horizon. In fact, there exists a limiting solution which consists of νN quarks with a flux tube that extends to infinity, and the brane approaches the horizon exponentially with distance.

From the above discussion it is clear that for large quark separation L the (renormalized) energy will receive its main contribution from the flux tube, and will consequently
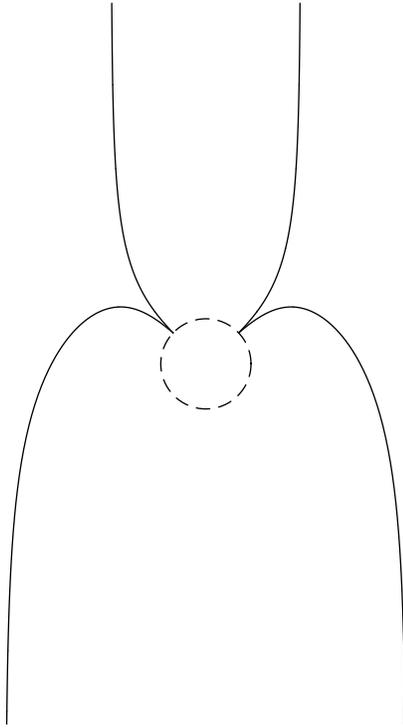
Figure 4: Polar plot of $r(\theta)$ – the two-dimensional projection of Fig. 3. The Born-Infeld string 'tubes' pointing up and down represent two groups of νN and $(1 - \nu)N$ quarks, respectively (with $\nu = 0.9$ here). The brane extends in the x-direction mostly at the inflection point (the cusp), while x is essentially constant along the tubes.

depend linearly on L, a clear indication of confinement. It is easy to compute the tension (energy per unit distance in x) of the color flux tube. Note that when the fivebrane runs parallel and close to the horizon, the energy function (4) reduces to

$$U_{flux} = \frac{2}{3\pi} NT_F \int dx \left(\frac{r}{R}\right)^2 \sqrt{D^2 + \sin^8 \theta} \ .$$

Using $r \simeq r_h$, $\theta \simeq \theta_c$, and performing some simple manipulations using the definitions of θ_c (10) and $D(\theta)$ (3), one obtains the tension

$$\sigma_3(\nu) = \frac{2}{3\pi} NT_F \left(\frac{r_h}{R}\right)^2 \sin^3 \theta_c = \frac{\sqrt{2}}{3} N \sqrt{g_{YM_3}^2 N} T^{3/2} \sin^3 \theta_c \ , \tag{12}$$

where the last expression is given solely in terms of parameters of the gauge theory in three dimensions. Since it is obtained by dimensional reduction from four dimensions, this theory should be understood to have an ultraviolet cutoff proportional to the Hawking temperature T. The dependence of the tension on T, N, and the 't Hooft coupling $\lambda_3 = g_{YM_3}^2 N$ agrees with the result of [19], where a baryon whose component quarks lie on a circle is treated within a simplified Nambu-Goto approach.

10

Notice that, in addition, equation (12) gives the dependence of the flux tube tension on ν, i.e. on its color content. This nontrivial dependence, arising entirely from the factor $\sin^3 \theta_c$, is plotted in Fig. 5:
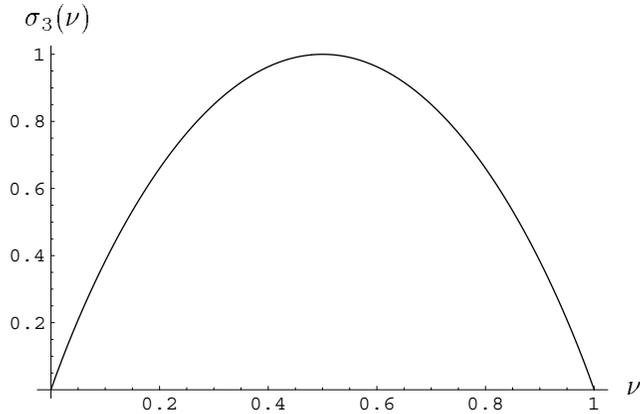

Figure 5: The tension of the flux tube (normalized to unity at its peak) as a function of ν, the fraction of quarks pulled apart. In the full theory ν should be quantized in units of $1/N$. See text for discussion.

Let us discuss the features of the flux tube tension seen in Fig. 5. As one might expect, the tension increases linearly for small ν. This means that for each additional quark removed the work done is approximately constant. Also as expected, in expression (12) there is a complete symmetry between ν and $1 - \nu$, (i.e., it makes no difference whether n or $N-n$ quarks are pulled out). Thus, the tension has a maximum at $\nu = 1/2$ and comes back down to zero near $\nu = 1$. In gauge theory language, the flat part of the curve means that it does not cost any energy to move the quark from one bundle with $\sim N/2$ quarks to the other.

Notice from equation (10) that $\sin^3 \theta_c \simeq 3\pi\nu/2$ for small ν. This implies that $\sigma_3(\nu = 1/N)$ becomes *independent* of N in the 't Hooft limit $N \to \infty$ with λ_3 fixed. This result has a natural gauge theory interpretation. When one quark is pulled out from the $SU(N)$ baryon (a color-singlet), the remaining $N - 1$ must be in the anti-fundamental representation of the gauge group. The flux tube extending between this bundle and the solitary quark should then have the same properties as the standard QCD string which connects a quark and an antiquark. In particular, its tension should depend on N only through the 't Hooft coupling, as we have found. As a matter of fact, for $\nu = 1/N \ll 1$ equation (12) precisely agrees with the quark-antiquark string tension which follows from a Nambu-Goto string calculation [11]. More generally, for $\nu = n/N$, with n fixed as $N \to \infty$, expression (12) reduces to the tension of n quark-antiquark strings.

It is important to note that, as has been pointed out by various authors, the gauge

11

theory under study here is not strictly three-dimensional [11, 5]. The energy scale associated with the QCD string tension, for instance, is proportional to $\lambda_4^{1/4}T$, where $\lambda_4 = g_{YM_4}^2 N$. This is much larger than the compactification scale T in the large λ_4 regime where the supergravity approximation is appropriate.

3 The Baryon in Four Dimensions

We are of course in principle more interested in baryons in four-dimensional gauge theories. One way of getting at this problem is via the dynamics of a D4-brane embedded in the background of a large number, N, of nonextremal D4-branes [5, 19, 21]. The argument is completely analogous to the D5-brane in a multiple D3-brane background that has been analysed in previous sections.

3.1 Worldvolume Action and Equations of Motion

The near-horizon metric and dilaton field in the D4-brane background are[9]

$$ds^2 = \left(\frac{r}{R}\right)^{3/2}\left[f(r)d\tau^2 + dx_{||}^2\right] + \left(\frac{R}{r}\right)^{3/2}f(r)^{-1}dr^2 + R^{3/2}\sqrt{r}d\Omega_4^2,$$

$$e^\phi = g_s\left(\frac{r}{R}\right)^{3/4},$$

$$f(r) = 1 - r_h^3/r^3, \qquad R^3 = \pi g_s N l_s^{\,3}, \qquad r_h = \frac{16\pi^2}{9}R^3 T^2,$$

(13)

where $\{\tau, x_{||}\} = \{\tau, x, y, z, w\}$ denote the directions parallel to the four-branes. The Hawking temperature T is also the inverse period of the Euclidean coordinate τ. The string coupling constant is determined by the dilaton in the original asymptotically flat region, $g_s = e^{\phi_\infty}$. The IIA supergravity solution (13) can only be trusted in the region where both the curvature in string units and the dilaton are small,

$$1 \ll R^3 r l_s^{\,-4} \ll N^{4/3}.$$

(14)

Type IIA string theory on this background (which can be understood as a compactification of an M5-brane system) is dual to $d = 4+1$ $SU(N)$ SYM theory (the infrared limit of the (0,2) theory on a circle) at temperature T, with coupling $g_{YM_5}^2 = (2\pi)^2 g_s l_s$ [22]. The gauge theory coordinates are $\{x, y, z, w, \tau\}$. For large T the τ circle becomes small and the theory is effectively a strongly-coupled four-dimensional Euclidean gauge theory at zero temperature, with coupling $g_{YM_4}^2 = g_{YM_5}^2 T$, expected to display behavior similar to that of non-supersymmetric pure Yang-Mills theory, QCD$_4$ [12, 5].

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[9]We thank N. Itzhaki for pointing out a numerical error in the expression for r_h that appeared in a previous version of this paper.

The baryon in the four-dimensional theory is dual to a string theory fourbrane on which N fundamental strings terminate [4, 5]. The worldvolume action for the four-brane is

$$S = -T_4 \int d^5\xi e^{-\tilde{\phi}} \sqrt{\det(g + F)} + T_4 \int A_{(1)} \wedge G_{(4)} \ ,$$

where $T_4 = 1/(g_s (2\pi)^4 l_s^{\ 5})$ is the brane tension, and $\tilde{\phi} = \phi - \phi_\infty$. The Wess-Zumino term couples the worldvolume gauge field $A_{(1)}$ to (the pullback of) the dual of the background six-form field strength $G_{(6)}$.

We assume a static (i.e. y-invariant) $SO(4)$ symmetric configuration of the form $r(\theta)$, $x(\theta)$ and $A_y(\theta)$ (with θ the polar angle of the $\mathbf{S}^4$). Using the explicit background (13) one can rewrite the Lagrangian (with a sign switch) in the form

$$U = T_4 \Omega_3 R^3 \int d\theta \sqrt{r^2 + f(r)^{-1}r'^2 + (r/R)^3 x'^2} \sqrt{D^2 + \sin^6\theta} \ , \tag{15}$$

where the displacement D now satisfies the equation

$$\partial_\theta D = -3\sin^3\theta,$$

and is consequently given by

$$D(\theta) = 3\cos\theta - \cos^3\theta - 2 + 4\nu \ . \tag{16}$$

The constant of integration has been written again in terms of a parameter $0 \leq \nu \leq 1$, which controls the number of Born-Infeld strings emerging from the D4-brane at each pole of the $\mathbf{S}^4$. For a given ν, it is easy to verify from (15) that the spikes at $\theta = \pi$ and $\theta = 0$ will have the same asymptotic 'tension' as $(1-\nu)N$ and νN fundamental strings, respectively. It is clear then that ν is quantized in units of $1/N$ in the full theory.

From (15) it is seen that if $\{r(\theta), x(\theta)\}$ is a solution for horizon radius r_h, then $\{\alpha r(\theta), \alpha^{-1/2} x(\theta)\}$ is a solution for horizon radius αr_h. Notice that this differs in functional form from what was seen in Section 2.1. The scaling $x \propto R^{3/2}/r^{1/2}$ is precisely the holographic scaling expected from the UV/IR relation in the near-horizon D4-brane background [17].

After repeating essentially the same routine as in the threebrane case, switching to the parametric representation $r = r(s)$, $x = x(s)$, $\theta = \theta(s)$ one finds the constraint Hamiltonian

$$2\tilde{H} = \left(1 - \frac{r_h^3}{r^3}\right) p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_x^2}{r^3} - \left(D^2(\theta) + \sin^6\theta\right) = 0 \ . \tag{17}$$

The equations of motion have the same basic structure as those obtained in the three-brane case:

13

$$\dot{r} = \left(1 - \frac{r_h^3}{r^3}\right)p_r , \qquad \dot{p}_r = \frac{3}{2r^4}(p_x^2 R^3 - p_r^2 r_h^3) + \frac{p_\theta^2}{r^3} ,$$

$$\dot{\theta} = \frac{p_\theta}{r^2} , \qquad \dot{p}_\theta = -6\sin^3\theta\,(2\nu - 1 + \cos\theta) , \qquad (18)$$

$$\dot{x} = \frac{R^3}{r^3}p_x, \qquad \dot{p}_x = 0 .$$

Again, for large r the Hamiltonian (17) reduces to that of a particle moving in two dimensions, this time in a potential $V(\theta) = -\left[D(\theta)^2 + \sin^6\theta\right]$. This potential has the same features as the one discussed in Section 3.1 and depicted in Fig. 1: two minima at $\theta = 0$ and $\theta = \pi$, and a maximum at $\theta = \theta_c$ such that $\cos\theta_c = 1 - 2\nu$.

3.2   Extremal Case: The BPS Baryon

We begin the analysis of the solutions by specializing to the extremal ($r_h = 0$) background, with the aim of obtaining the supersymmetric fourbrane embeddings analogous to the fivebrane BPS configurations found in [6]. Following that paper, we set $x' = 0$ in (15), to describe a baryon of zero-size. The Euler-Lagrange equation for $r(\theta)$ is then

$$\frac{d}{d\theta}\left(\frac{r'}{\sqrt{r^2 + r'^2}}\sqrt{D^2 + \sin^6\theta}\right) = \frac{r}{\sqrt{r^2 + r'^2}}\sqrt{D^2 + \sin^6\theta} . \qquad (19)$$

The system under consideration here is in such close correspondence to the one studied in [6], that it is easy to guess the BPS condition analogous to the one for a fivebrane embedded in $AdS_5 \times \mathbf{S}^5$ [7, 6, 23]:

$$\frac{r'}{r} = \frac{\sin^4\theta + D(\theta)\cos\theta}{\sin^3\theta\cos\theta - D(\theta)\sin\theta} , \qquad (20)$$

where $D(\theta)$ is the function specified in (16). It is easy to verify that (20) is a first integral of (19). It is thus almost certainly the condition for the fourbrane embedding to be supersymmetric, although we have not checked this explicitly.

Just like its fivebrane cousin [6], equation (20) can be solved analytically to obtain a one-parameter family of BPS configurations,

$$r(\theta) = \frac{A}{\sin\theta}\left[\frac{\eta(\theta)}{2(1-\nu)}\right]^{1/2} , \qquad \eta(\theta) = 1 - 2\nu - \cos\theta, \qquad (21)$$

where the scale factor A is arbitrary, and ν is the integration constant in (16). The solutions (21) have the exact same features as their fivebrane counterparts (see Eq. (9) and Fig. 1 in [6]): they describe 'tubes' of radius A ($r \sim A/(\pi - \theta)$ when $\theta \to \pi$), with an asymptotic 'tension' and total energy equal to those of $(1 - \nu)N$ strings. The

14

$\nu = 0$ embedding has the shape of a 'test tube' with $r(0) = r_0 = A/2$. This solution captures all N units of fourform flux; it corresponds to a baryon in the $d = 4+1$ SYM theory. Notice that the surfaces (21) are defined in the range where $\eta(\theta) \geq 0$, namely $\theta_c \leq \theta \leq \pi$, where the critical angle θ_c is defined by

$$\cos \theta_c = 1 - 2\nu. \tag{22}$$

For $\nu > 0$, the solutions intersect $r = 0$ at $\theta = \theta_c > 0$, and capture only a fraction of the total flux. They represent gauge theory objects with fewer than N quarks (see the discussion in [6]).

3.3 Nonextremal Case: The Point Baryon

Having understood the character of the BPS configurations in the extremal background, we now proceed to examine the solutions for finite horizon radius. The fourbrane embeddings corresponding to baryons of the four-dimensional gauge theory with all of their component quarks at the same gauge theory position ($x = 0$) and $\mathbf{S}^5$ angle ($\theta = \pi$), have $p_x = 0$ (i.e. $x' = 0$) and $\nu = 0$ (see Section 3.4 for the case $x' \neq 0$, $\nu > 0$). As in Section 2.2, to obtain a smooth solution we enforce the boundary conditions $\partial_\theta r = 0$ and $r = r_0$ at $\theta = 0$.

Numerical integration with these boundary conditions yields results completely analogous to those found for the three-dimensional case in Section 2.2. There exists a one-parameter family of solutions with parameter $\zeta = r_0/r_h \geq 1$ (see Fig. 2). When $\zeta < \zeta_{crit} \approx 18$ the solution tilts towards and eventually crosses the symmetry axis, thus reaching $\theta = \pi$ at a finite value of r. As $\zeta \to 1$ (i.e. $r_0 \to r_h$), the brane becomes more and more spherical (notice that $r(\theta) = r_h$ is a solution to the equations of motion). For $\zeta > \zeta_{crit}$, on the other hand, the solution tilts away from the symmetry axis, reaching $\theta = \pi$ only at $r = \infty$.

The cusp in the $\zeta < \zeta_{crit}$ solutions is again to be understood as the point of insertion of N fundamental strings (or equivalently, a collapsed fourbrane), running along the ray $\theta = \pi$. A variational calculation like the one performed in Section 2.2 shows that the stable configuration is the one with $\zeta \to \infty$. So again the embeddings (similar to those portrayed in Fig. 2) describe the process through which the N fundamental strings deform the initially spherical fourbrane, pulling it out to infinity. The final configuration has the shape of a 'tube', just like the BPS embeddings found in the previous subsection. In the supersymmetric case r_0 is a modulus of the configuration; but in the $r_h > 0$ case a potential drives the stable solution away to infinity.

3.4 The Split Baryon: Color Dependence of the String Tension

In this section we study solutions with $p_x \neq 0$ (i.e. $x' \neq 0$) and $\nu > 0$, which correspond to a baryon split into two separate groups of νN and $(1 - \nu)N$ quarks, a distance L apart, and connected by a color flux tube. The story here is in perfect parallel to the three-dimensional case discussed in Section 2.3, although the quantitative results are slightly different. The two groups of quarks are represented by Born-Infeld strings bundles protruding from opposite poles of the $\mathbf{S}^4$, $\theta \to 0$ and $\theta \to \pi$. The flux tube connecting the two bundles of quarks is dual to a segment of the fourbrane that runs close and parallel to the horizon, at the critical angle $\theta = \theta_c$ given by equation (22). This is the maximum of the potential introduced at the end of Section 3.1. The critical angle as a function of ν has the property that $\theta_c(0) = 0$, and $\theta_c(\nu) = \pi - \theta_c(1 - \nu)$.

Numerical integration with initial conditions analogous to (11) yields fourbrane embeddings of the same type as those found in the three-dimensional case (see Figs. 3 and 4). For large quark separation L the energy of the configuration is again proportional to L, indicating confinement. Due to the fact that the relation between ν and θ_c is now simpler than that in Section 2.3, the tension of the flux tube can now be given very explicitly as a function of ν. It is

$$\sigma_4(\nu) = \frac{8\pi}{27} N(g^2_{YM_4} N) T^2 \nu (1 - \nu) \ . \tag{23}$$

The different powers of T, N, and the 't Hooft coupling $\lambda_4 = g^2_{YM_4} N$ match those found in [5, 19, 21], where the baryon is studied within a somewhat different scheme.

The simple dependence of the flux tube tension on ν is perhaps what one would have naively guessed: the 'product of the charges'. It is amusing to see that this differs only slightly from the three-dimensional result. For ease of comparison the ν-dependence of the tension (normalized to unity at the maximum) is plotted for both cases in Fig. 6.

All the qualitative features of the tension as a function of ν are the same as in the three-dimensional gauge theory. In particular, for $\nu = 1/N \ll 1$ (i.e., when a single quark is pulled out), σ_4 is independent of N (for given 't Hooft coupling) and exactly matches the quark-antiquark string tension obtained in [11].[10]

Finally, we want to emphasize that again the excitation energies associated with the flux tube tension (23) in the supergravity regime are larger than the compactification scale, so the gauge theory is not strictly four-dimensional [5, 11].

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[10]Notice that equation (3.20) in [11] is incorrect by a factor of π^3.

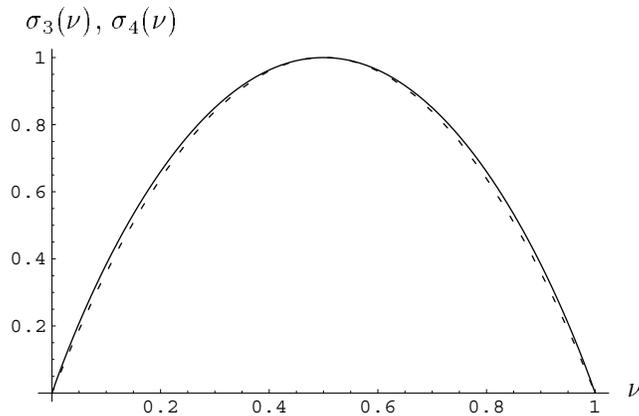Figure 6: The tension of the flux tubes of the three- and four-dimensional split baryon (solid and dashed curve, respectively), as a function of ν, the fraction of separated quarks. See text for discussion.

4   Conclusions

We have seen how studying the detailed shape of a D5/4-brane, embedded in the near-horizon geometry of a large number N of non-extremal D3/4-branes, can reveal much information about the structure and energetics of baryons in the corresponding strongly-coupled $SU(N)$ gauge theory. In particular, we were able to construct embeddings representing a baryon split into two separate clumps of quarks, with a color flux tube running between them. As expected for a confining theory, the energy of such a configuration is proportional to the separation between the two quark bundles. Reading off the tension of the flux tube, we discovered a non-trivial and physically reasonable dependence on the color charges of the individual clumps.

We also studied the simpler 'point-baryon' embeddings, where all the quarks sit on top of each other. The use of the full Born-Infeld plus Wess-Zumino action for the system makes it clear that the naive picture of the baryon as N fundamental strings terminating on a spherical five/four-brane is incomplete: the strings pull on the brane and deform it. Our solutions displayed several interesting features, such as the appearance of a potential for the overall scale of the system (which was a modulus in the extremal case), that ultimately causes the brane to expand out to infinity. Many of the features in these solutions call out for a deeper explanation. In particular, they point to the need for a fuller understanding of the relation between the angular (S^5/S^4) dependence of the D5/4-brane embedding and the flavor structure and energetics of the dual baryonic state.

An obvious extension of our work would be to study the baryon in thermal gauge theories. For the example of finite temperature (3+1)-dimensional SYM, one considers

17

again a fivebrane embedded in the nonextremal D3-brane geometry (1), but this time extending along the compact τ direction. A study of this system leads to some puzzling features, which we hope to clarify and report on in the near future.

Recently, Type 0 string theory has been suggested as a means to construct gravity duals for large N non-supersymmetric four-dimensional gauge theories with phenomenologically interesting behavior [13, 14]. In particular, it has been argued that, at least at the classical level, the Type 0 equations of motion have asymptotic solutions which express asymptotic freedom in the UV and confinement in the IR [14]. It would be very interesting to study the baryon in this setting. At present, the full background geometry is not known, so the analysis would have to be restricted to fivebrane embeddings which lie only in the asymptotic UV region. We hope that a complete study will eventually become possible.

In our analysis the Born-Infeld string approach has again proven very fruitful. We expect that many more lessons will be extracted with this valuable tool in the future.

Acknowledgements

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